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Chapter 20

LINEAR INDUCTION MOTORS

20.1 INTRODUCTION

For virtually every rotary electric machine, there is a linear motion counterpart. So is the case with induction machines. They are called linear induction machines (LIMs).

LIMs directly develop an electromagnetic force, called *thrust*, along the direction of the travelling field motion in the airgap.

The imaginary process of “cutting” and “unrolling” rotary counterpart is illustrated in [Figure 20.1](#).

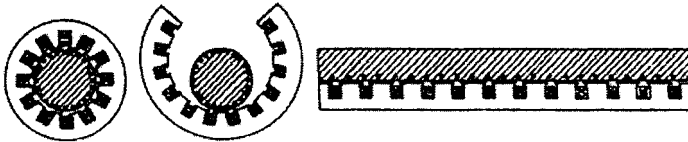


Figure 20.1 Imaginary process of obtaining a LIM from its rotary counterpart

The *primary* usually contains a three phase winding in the uniform slots of the laminated core.

The *secondary* is either made of a laminated core with a ladder cage in the slots or of an aluminum (copper) sheet with (or without) a solid iron back core.

Apparently the LIM operates as its rotary counterpart does, with *thrust* instead of torque and linear speed instead of angular speed, based on the principle of travelling field in the airgap.

In reality there are quite a few differences between linear and rotary IMs such as [1 - 8]

- The magnetic circuit is open at the two longitudinal ends (along the travelling field direction). As the flux law has to be observed, the airgap field will contain additional waves whose negative influence on performance is called *dynamic longitudinal end effect* ([Figure 20.2a](#)).
- In short primaries (with 2, 4 poles), there are current asymmetries between phases due to the fact that one phase has a position to the core longitudinal ends which is different from those of the other two. This is called static longitudinal effect ([Figure 20.2b](#)).
- Due to same limited primary core length, the back iron flux density tends to include an additional nontravelling (ac) component which should be considered when sizing the back iron of LIMs ([Figure 20.2c](#)).
- In the LIM on [Figure 20.1](#) (called single sided, as there is only one primary along one side of secondary), there is a normal force (of attraction or

repulsion type) between the primary and secondary. This normal force may be put to use to compensate for part of the weight of the moving primary and thus reduce the wheel wearing and noise level (Figure 20.2d).

- For secondaries with aluminum (copper) sheet with (without) solid back iron, the induced currents (in general at slip frequency Sf_1) have part of their closed paths contained in the active (primary core) zone (Figure 20.2c). They have additional-longitudinal (along OX axis)-components which produce additional losses in the secondary and a distortion in the airgap flux density along the transverse direction (OY). This is called the transverse edge effect.

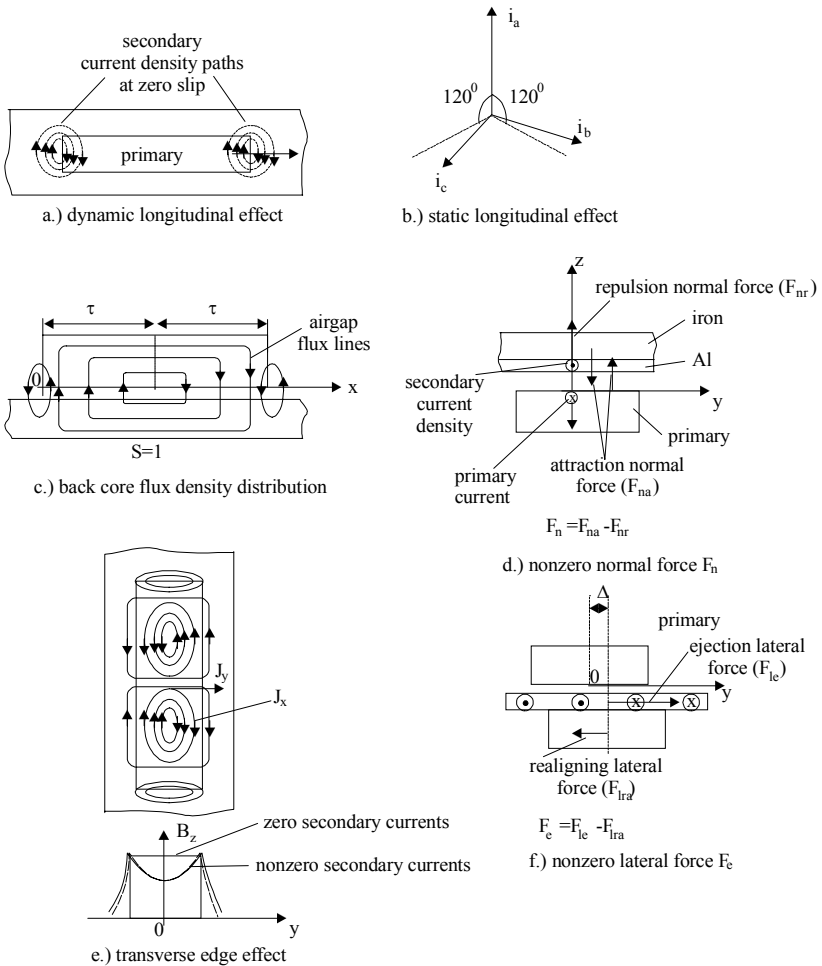


Figure 20.2 Panoramic view of main differences between LIMs and rotary IMs

- When the primary is placed off center along OY, the longitudinal components of the current density in the active zone produce an ejection

type lateral force. At the same time, the secondary back core tends to realign the primary along OY. So the resultant lateral force may be either decentralizing or centralizing in character (Figure 20.2.f).

All these differences between linear and rotary IMs warrant a specialized investigation of field distribution and performance in order to limit the adverse effects (longitudinal end effects and back iron flux distortion, etc.) and exploit the desirable ones (normal and lateral forces, or transverse edge effects).

The same differences suggest the main merits and demerits of LIMs.

Merits

- Direct electromagnetic thrust propulsion (no mechanical transmission or wheel adhesion limitation for propulsion)
- Ruggedness; very low maintenance costs
- Easy topological adaptation to direct linear motion applications
- Precision linear positioning (no play (backlash) as with any mechanical transmission)
- Separate cooling of primary and secondary
- All advanced drive technologies for rotary IMs may be applied without notable changes to LIMs

Demerits

- Due to large airgap to pole pitch (g/τ) ratios— $g/\tau > 1/250$ —the power factor and efficiency tend to be lower than with rotary IMs. However, the efficiency is to be compared with the combined efficiency of rotary motor + mechanical transmission counterpart. Larger mechanical clearance is required for medium and high speeds above 3m/s. The aluminum sheet (if any) in the secondary contributes an additional (magnetic) airgap.
- Efficiency and power factor are further reduced by longitudinal end effects. Fortunately these effects are notable only in high speed low pole count LIMs and they may be somewhat limited by pertinent design measures.
- Additional noise and vibration due to uncompensated normal force, unless the latter is put to use to suspend the mover (partially or totally) by adequate close loop control.

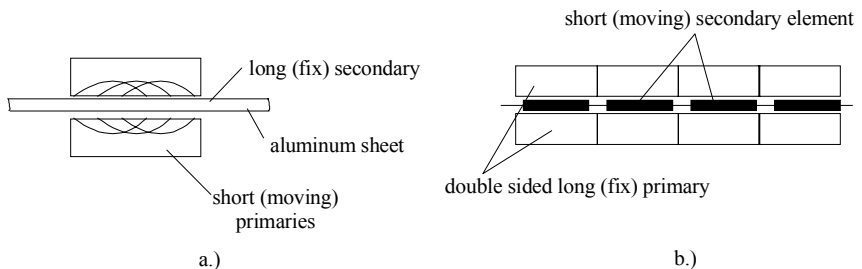


Figure 20.3 Double sided flat LIMs

a.) double sided short (moving) primary LIM; b.) double sided short (moving) secondary LIM for conveyors

As sample LIM applications have been presented in Chapter 1, we may now proceed with the investigation of LIMs, starting with classification and practical construction aspects.

20.2 CLASSIFICATIONS AND BASIC TOPOLOGIES

LIMs may be built single sided (Figure 20.1) or double sided (Figure 20.3a), with moving (short) primary (Figure 20.1) or moving (short) secondary (Figure 20.3b).

As single sided LIMs are more rugged, they have found more applications. However (Figure 20.3b) shows a double sided practical short-moving secondary LIM for low speed short travel applications.

LIMs on Figure 20.1-20.3 are flat. For flat single sided LIMs the secondaries may be made of aluminum (copper) sheets on back solid iron (for low costs), ladder conductor in slots of laminated core (for better performance), and a pure conducting layer in electromagnetic metal stirrers (Figure 20.4a, b, c).

In double sided LIMs, the secondary is made of an aluminum sheet (or structure) or from a liquid metal (sodium) as in flat LIM pumps.

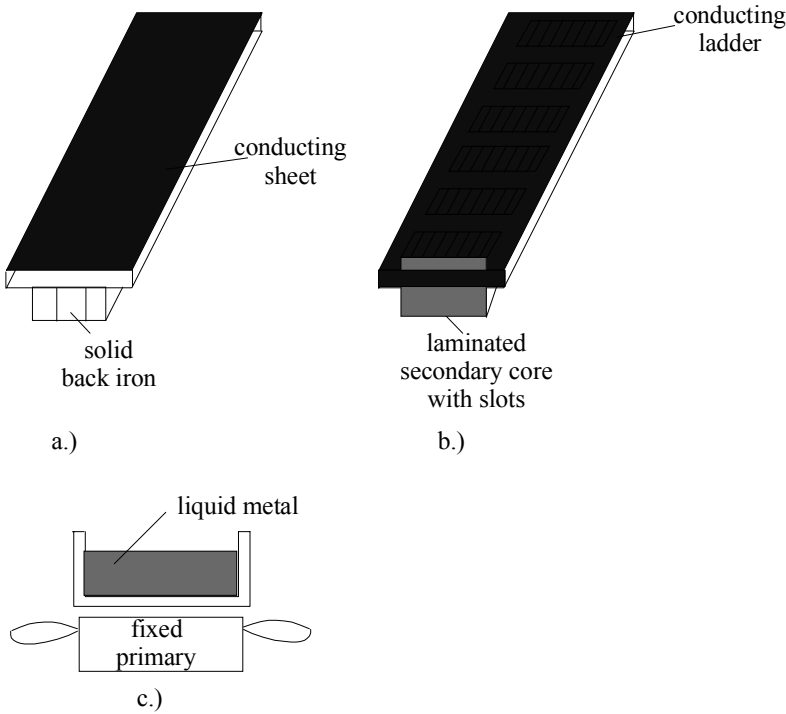


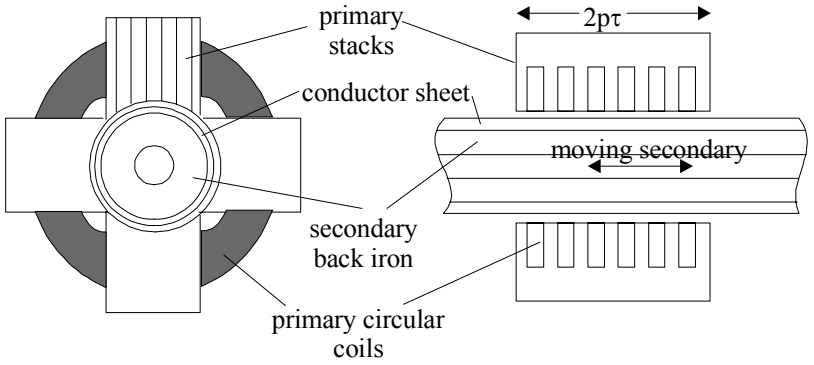
Figure 20.4 Flat single-sided secondaries
a.) sheet on iron ; b.) ladder conductor in slots; c.) liquid metal

Besides flat LIMs, tubular configurations may be obtained by rerolling the flat structures along the transverse (OY) direction (Figure 20.5). Tubular LIMs or tubular LIM pumps are in general single sided and have short fix primaries and moving limited length secondaries (except for liquid metal pumps).

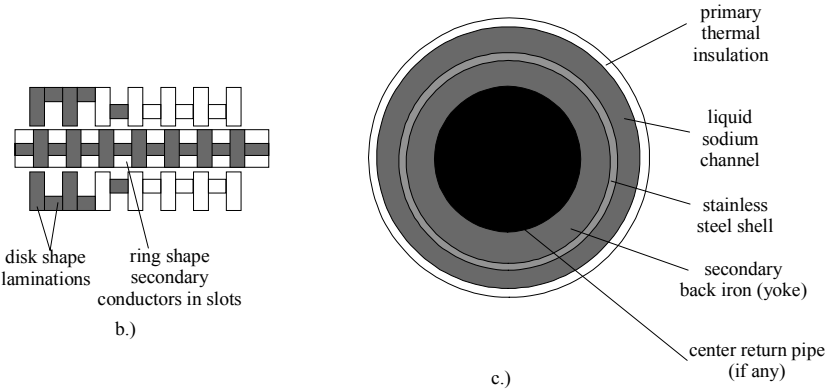
The primary core may be made of a few straight stacks (Figure 20.5a) with laminations machined to circular stator bore shape. The secondary is typical aluminum (copper) sheet on iron. The stator coils have a ring shape.

While transverse edge effect is absent and coils appear to lack end connections, building a well centered primary is not easy.

An easier solution to build is obtained with only two-size disk shape laminations both on primary and secondary (Figure 20.5b). The secondary ring shape conductors are also placed in slots.



a.)



c.)

Figure 20.5 Tubular LIMs

- a.) with longitudinal primary lamination stacks; b.) with disk-shape laminations
- c.) liquid metal tubular LIM pump the secondary

Better performance is expected by the fact that, in the back cores, the magnetic field goes perpendicular to laminations and tends to produce

additional core losses. The interlamination insulation leads to an increased magnetization m.m.f. and thus takes back part of this notable improvement.

The same rationale is valid for tubular LIM liquid metal pumps (Figure 20.5c) in terms of primary manufacturing process. Pumps allow for notably higher speeds ($u = 15\text{m/s}$ or more) when the fixed primaries may be longer and have more poles ($2p_1 = 8$ or more). The liquid metal (sodium) low electrical conductivity leads to a smaller dynamic longitudinal effect which at least has to be checked to see if it is negligible.

20.3 PRIMARY WINDINGS

In general, three phase windings are used as three phase PWM converters and are widely available for rotary induction motor drives. Special applications which require only $2p_1 = 2$ pole might benefit from two phase windings as they are both placed in the same position with respect to the magnetic core ends. Consequently, the phase currents are fully symmetric at very low speeds.

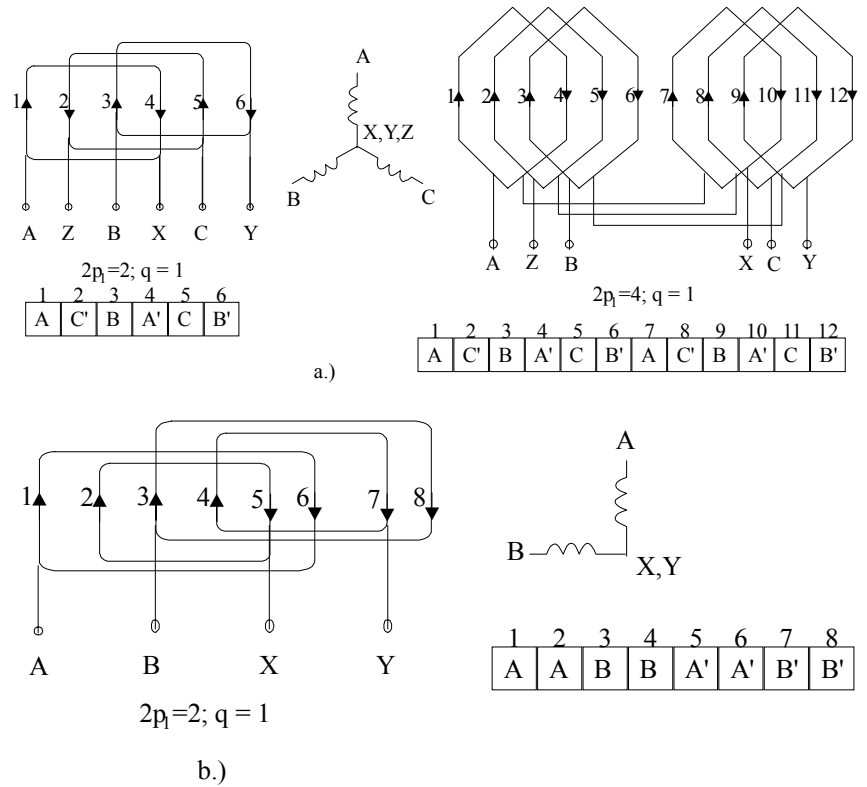


Figure 20.6 Single layer windings
a.) three phase; b.) two phase

A			A'	A'		A			A'	A'		A
B'	B		B		B'			B			B'	
	C		C	C	C	C'	C'		C	C	C	C'

Figure 20.7 Triple layer winding with chorded coils ($y/\tau = 2/3$) (very short end connections)

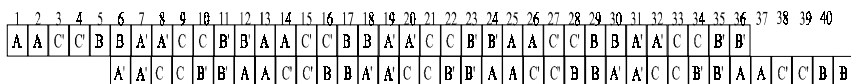


Figure 20.8 Double layer chorded coil windings with $2p_1 + 1$ poles

- Single layer full pitch ($y = \tau$) windings with an even number of poles $2p_1$, [Figure 20.6](#) three phase and two phase.
- Triple layer chorded coil ($y/\tau = 2/3$) winding with an even number of poles $2p_1$ ([Figure 20.7](#)).
- Double layer chorded coil ($2/3 < y/\tau < 1$) coil winding with an odd number of poles $2p_1 + 1$ (the two end poles have half-filled slots)-[Figure 20.8](#).
- Fractionary winding for miniature LIMs ([Figure 20.9](#)).

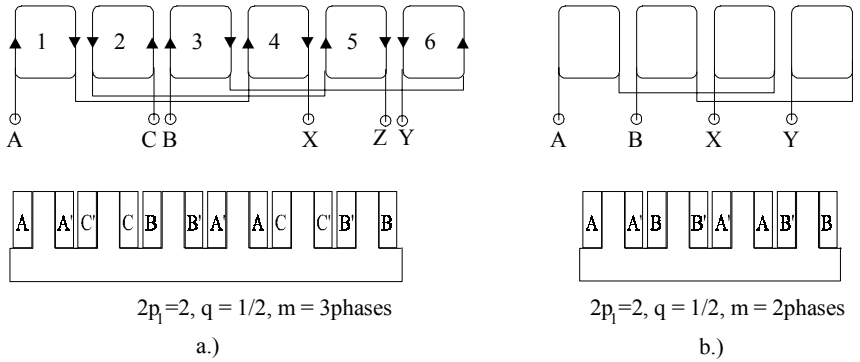


Figure 20.9 Fractionary single layer winding (low low end connections)
 a.) three phase; b.) two phase

A few remarks are in order

- The single layer winding with an even number of poles makes better usage of primary magnetic core but it shows rather large coil end connections. It is recommended for $2p_1 = 2, 4$.
- The triple layer chorded coil windings is easy to manufacture automatically; it has low end connections but it also has a rather low winding (chording) factor $K_y = 0.867$.
- The double layer chorded coil winding with an odd total number of poles (Figure 20.8) has shorter end connections and is easier to build but it makes a poorer use of primary magnetic core as the two end poles are halfwound. As the number of poles increases above 7, 9 the end poles influence becomes small. It is recommended for large LIMs ($2p_1 + 1 > 5$).
- The fractionary winding (Figure 20.9)-with $q = \frac{1}{2}$ in our case-is characterised by very short end connections but the winding factor is low. It is recommended only in miniature LIMs where volume is crucial.
- When the number of poles is small, $2p_1 = 2$ especially, and phase current symmetry is crucial (low vibration and noise) the two phase LIM may prove the adequate solution.
- Tubular LIMs are particularly suitable for single-layer even-number of poles windings as the end connections are nonexistent with ring-shape coils.

In the introduction we mentioned the transverse edge and longitudinal end effects as typical to LIMs. Let us now proceed with a separate analysis of transverse edge effect in double sided and in single sided LIMs with sheet-secondary.

20.4 TRANSVERSE EDGE EFFECT IN DOUBLE-SIDED LIM

A simplified single dimensional theory of transverse edge effect is presented here.

The main assumptions are

- The stator slotting is considered only through Carter coefficient K_c .

$$K_c = \frac{1}{1 - \gamma g / \tau_s}; \quad \gamma = \frac{\left(\frac{g}{b_{os}}\right)^2}{5 + \gamma \frac{g}{b_{os}}} \quad (20.1)$$

- The primary winding in slots is replaced by an infinitely thin current sheet traveling wave $J_1(x, t)$

$$J_1(x, t) = J_m e^{j\left(S\omega_1 t - \frac{\pi}{\tau} x\right)}, \quad J_m = \frac{3\sqrt{2}W_1 K_{w1} I_1}{p_1 \tau} \quad (20.2)$$

$2p_1$ -pole number, W_1 -turns/phase, K_{w1} -winding factor; τ -pole pitch, I_1 -phase current (RMS). Coordinates are attached to secondary.

- The skin effect is neglected or considered through the standard correction coefficient.

$$K_{skin} \approx \frac{d}{2d_s} \left[\frac{\sinh\left(\frac{d}{d_s}\right) + \sin\left(\frac{d}{d_s}\right)}{\cosh\left(\frac{d}{d_s}\right) - \cos\left(\frac{d}{d_s}\right)} \right] \quad (20.3)$$

$$\frac{1}{d_s} \approx \sqrt{\frac{\mu_0 \omega_1 S \sigma_{Al}}{2}} \quad (20.4)$$

For single sided LIMs, d/d_s will replace $2d/d_s$ in (20.2); d_s -skin depth in the aluminum (copper) sheet layer. Consequently, the aluminum conductivity is corrected by $1/K_{skin}$,

$$\sigma_{Als} = \frac{\sigma_{Al}}{K_{skin}} \quad (20.5)$$

- For a large airgap between the two primaries, there is a kind of flux leakage which makes the airgap look larger g_l [1].

$$g_l = g K_c K_{leakage}$$

$$K_{leakage} = \frac{\sinh\left(\frac{g}{2\tau}\right)}{\frac{g}{2\tau}} > 1 \quad (20.6)$$

- Only for large g/τ ratio K_{leakage} is notably different from unity. The airgap flux density distribution in the absence of secondary shows the transverse fringing effect (Figure 20.10).

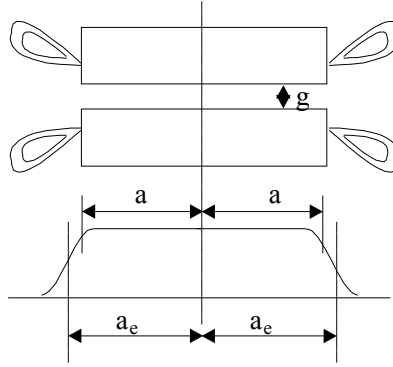


Figure 20.10 Fringing and end connection flux considerations

The transverse fringing effect may be accounted for by introducing a larger (equivalent) stack width $2a_e$ instead of $2a$

$$2a_e = 2a + (1.2 \div 2.0) g \quad (20.7)$$

For large airgap in low thrust LIMs this effect is notable.

As expected, the above approximations may have been eliminated provided a 3D FEM model was used. The amount of computation effort for a 3D FEM model is so large that it is feasible mainly for special cases rather than for preliminary or optimisation design.

- Finally, the longitudinal effect is neglected for the time being and space variations along thrust direction and time variations are assumed to be sinusoidal.

In the active region ($|z| \leq a_e$) Ampere's law along contour 1 (Figure 20.11) yields

$$g_e \frac{\partial \underline{H}_y}{\partial z} = \underline{J}_{2x} d; \quad \underline{J}_1 = \underline{J}_m e^{j\left(\omega_1 t - \frac{\pi}{\tau} x\right)} \quad (20.8)$$

Figure 20.11 shows the active and overhang regions with current density along motion direction.

The same law applied along contour 2, Figure 20.11.b, in the longitudinal plane gives

$$-g_e \frac{\partial}{\partial x} (\underline{H}_x + \underline{H}_0) = \underline{J}_m + \underline{J}_{2z} d \quad (20.9)$$

Faraday's law also yields

$$\frac{\partial J_{2z}}{\partial x} - \frac{\partial J_{2x}}{\partial z} = -j\mu_0 S\omega_1 \sigma_{\text{Als}} (\underline{H}_y + \underline{H}_0) \quad (20.10)$$

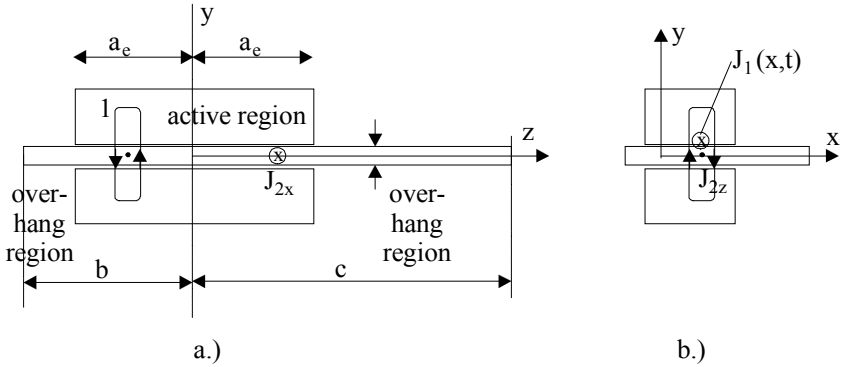


Figure 20.11 Transverse cross-section of a double sided LIM a.), and longitudinal view b.)

Equations (20.8-20.10) are all in complex terms as sinusoidal time variation was assumed

$\underline{H}_0$ is the airgap field in absence of secondary

$$\underline{H}_0 = \frac{jJ_m}{\frac{\pi}{\tau} g_e} \quad (20.11)$$

The three equations above combine to yield

$$\frac{\partial^2 \underline{H}_y}{\partial x^2} + \frac{\partial^2 \underline{H}_y}{\partial z^2} - j\mu_0 S\omega_1 \sigma_{\text{Als}} \frac{d}{g_e} \underline{H}_y = j\mu_0 S\omega_1 \sigma_{\text{Als}} \frac{d}{g_e} \underline{H}_0 \quad (20.12)$$

The solution of (20.12) becomes

$$\underline{H}_y = -\frac{j\underline{H}_0 S G_e}{1 + jS G_e} + A \cosh \underline{\alpha} z + B \sinh \underline{\alpha} z \quad (20.13)$$

with

$$G_i = \frac{\tau^2 \omega_1 \mu_0 \sigma_{\text{Als}}}{\pi^2} \frac{d}{g_e} \quad (20.14)$$

$$\underline{\alpha}^2 = \left(\frac{\pi}{\tau} \right)^2 (1 + jsG_e) \quad (20.15)$$

G_i is called the rather ideal goodness factor of LIM, a performance index we will return to in what follows frequently.

In the overhang region ($|z| > a_e$) we assume that the total field is zero, that is J_{2z} , J_{2x} satisfy Laplace's equation

$$\frac{\partial J_{2z}}{\partial x} - \frac{\partial J_{2x}}{\partial z} = 0 \quad (20.16)$$

Consequently

$$J_{2zr} = \underline{C} \sinh \frac{\pi}{\tau} (c - z) \quad a_e < z \leq c \quad (20.17)$$

$$J_{2xr} = j\underline{C} \cosh \frac{\pi}{\tau} (c - z)$$

$$J_{2zl} = \underline{D} \sinh \frac{\pi}{\tau} (b + z) \quad -b < z < -a_e \quad (20.18)$$

$$J_{2xl} = j\underline{D} \cosh \frac{\pi}{\tau} (b + z)$$

where r and l refer to right and left, respectively.

From the continuity boundary conditions at $z = \pm a_e$, we find

$$\underline{A} = \frac{H_0 j S G_i}{1 + j S G_i} \left[\frac{(\underline{C}_1 + \underline{C}_2) \cosh(\underline{\alpha} a_e) + 2 \sinh(\underline{\alpha} a_e)}{(1 + \underline{C}_1 \underline{C}_2) \sinh(2\underline{\alpha} a_e) + (\underline{C}_1 + \underline{C}_2) \cosh(2\underline{\alpha} a_e)} \right] \quad (20.19)$$

$$\underline{B} = \frac{\underline{A}(\underline{C}_2 - \underline{C}_1) \sinh(\underline{\alpha} a_e)}{(\underline{C}_1 + \underline{C}_2) \cosh(\underline{\alpha} a_e) + 2 \sinh(\underline{\alpha} a_e)}$$

$$\underline{C}_1 = \frac{\alpha \tau}{\pi} \tanh \frac{\pi}{\tau} (c - a_e); \quad \underline{C}_2 = \frac{\alpha \tau}{\pi} \tanh \frac{\pi}{\tau} (b - a_e) \quad (20.20)$$

$$\underline{C} = \frac{-j g_e \underline{\alpha}}{d \cosh \frac{\pi}{\tau} (c - a_e)} [\underline{A} \sinh(\underline{\alpha} a_e) + \underline{B} \cosh(\underline{\alpha} a_e)] \quad (20.21)$$

$$\underline{D} = \frac{-j g_e \underline{\alpha}}{d \cosh \frac{\pi}{\tau} (b - a_e)} [-\underline{A} \sinh(\underline{\alpha} a_e) + \underline{B} \cosh(\underline{\alpha} a_e)] \quad (20.22)$$

Sample computation results of flux and current densities distributions for a rather high speed LIM with the data of $\tau = 0.35\text{m}$, $f_1 = 173.3\text{Hz}$, $S = 0.08$, $d = 6.25\text{mm}$, $g = 37.5\text{mm}$, $J_m = 2.25 \cdot 10^5 \text{A/m}$ are shown in [Figure 20.12](#).

$r = 0.35 \text{ m}$
 $f = 173.3 \text{ Hz}$
 $s = 0.08$
 $d = 6.25 \text{ mm}$
 $g = 37.5 \text{ mm}$
 $J_m = 2.25 \times 10^5 \text{ A/m}$

$B_y \text{ [T]}$
 $J_{2z} \text{ (A/mm}^2\text{)}$
 J_{2x}

$|B_y|$
 $|J_{2z}|$
 $|J_{2x}|$

Secondary sheet
 Primaries

$a_p = 0.15 \text{ m}$
 a_p
 $b = 0.3 \text{ m}$
 $c = 0.2 \text{ m}$

Graph showing Lateral decentralizing force (N) versus $b-c$ (m) for $2p_l=12$ poles. The force increases non-linearly with distance.

The main consequences of transverse edge effect are an apparent increase in the secondary equivalent resistance R_2' and a decrease in the magnetisation

inductance (reactance X_m). Besides, when the secondary is off center in the transverse (lateral) direction, a lateral decentralising force F_{za} is produced

$$F_{za} = \mu_0 d \tau p_1 \operatorname{Re} \int_{-a_e}^{a_e} (\underline{J}_{2x} H_y^*) dz \quad (20.23)$$

Again sample results for the same LIM as above are given in [Figure 20.13](#).

The lateral force F_z decreases as a_e/τ decreases or the overhangs $c-a_e$, $b-a_e > \tau/\pi$. In fact it is of no use to extend the overhangs of secondary beyond τ/π as there are few currents for $|z| > |\tau/\pi + a_e|$.

The transverse edge effect correction coefficients

In the absence of transverse edge effect, the magnetization reactance X_m has the conventional expression (for rotary induction machines)

$$X_m = \frac{6\mu_0\omega_l}{\pi^2} \frac{(W_l K_{wl})^2 \tau (2a_e)}{p_l g_l} \quad (20.24)$$

The secondary resistance reduced to the primary R'_2 is

$$R'_2 = \frac{X_m}{G_i} = \frac{12(W_l K_{wl})^2 a_e}{\sigma_{Als} p_l \tau d} \quad (20.25)$$

Because of the transverse edge effect, the secondary resistance is increased by $K_t > 1$ times and the magnetization inductance (reactance) is decreased by $K_m < 1$ times.

$$K_t = \frac{K_X^2}{K_R} \left[\frac{1 + S^2 G_i^2 K_R^2 / K_X^2}{1 + S^2 G_i^2} \right] \geq 1 \quad (20.26)$$

$$K_m = \frac{K_R K_t}{K_X} \leq 1 \quad (20.27)$$

For $b = c$ [9],
$$K_R = 1 - \operatorname{Re} \left[(1 - SG_i) \frac{\lambda}{\underline{\alpha} a_e} \tanh(\underline{\alpha} a_e) \right] \quad (20.28)$$

$$K_X = 1 + \operatorname{Re} \left[(j + SG_i) \frac{SG_i \lambda}{\underline{\alpha} a_e} \tanh(\underline{\alpha} a_e) \right] \quad (20.29)$$

$$\underline{\lambda} = \frac{1}{1 + (1 + jSG_i)^{1/2} \tanh(\underline{\alpha} a_e) \cdot \tanh \frac{\pi}{\tau} (c - a_e)} \quad (20.30)$$

For LIMs with narrow primaries ($2a_e/\tau < 0.3$) and at low slips, $K_m \approx 1$ and

$$K_t \approx \frac{1}{1 - \frac{\tanh \frac{\pi}{\tau} a_e}{\frac{\pi}{\tau} a_e} \left[1 + \tanh \left(\frac{\pi}{\tau} a_e \right) \cdot \tanh \frac{\pi}{\tau} (c - a_e) \right]^{-1}} \quad (20.31)$$

Transverse edge effect correction coefficients depend on the goodness factor G_i , slip, S , and the geometrical type factors a_e/τ , $(c - a_e)/\tau$.

For $b \neq c$ the correction coefficients have slightly different expressions but they may be eventually developed based on the flux and secondary current densities transverse distribution.

The transverse edge effect may be exploited for developing large thrusts with lower currents or may be reduced by large overhangs (up to τ/π) and an optimum a_e/τ ratio.

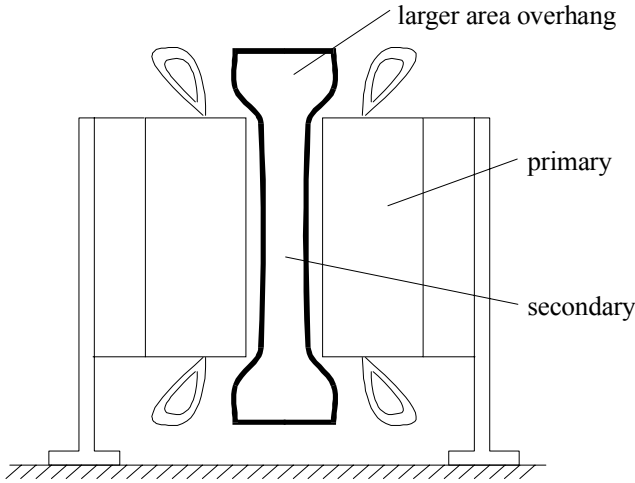


Figure 20.14 Reduced transverse edge effect secondary

On the other hand, the transverse edge effect may be reduced, when needed, by making the overhangs of a larger cross-section or of copper (Figure 20.14).

In general, the larger the value of SG_i , the larger the transverse edge effect for given a_e/τ and c/τ . In low thrust (speed) LIMs as the pole pitch τ is small, so is the synchronous speed u_s

$$u_s = 2\tau f_1 = \tau \frac{\omega_l}{\pi} \quad (20.32)$$

Consequently, the goodness factor G_i is rather small below or slightly over unity.

Now, we may define an equivalent-realistic-goodness factor G_e as

$$G_e = G_i \frac{K_m}{K_t} < G_i \quad (20.33)$$

where the transverse edge effect is also considered.

Alternatively, the combined airgap leakage (K_{leak}), skin effect (K_{skin}) and transverse edge effect (K_m , K_t) may be considered as correction coefficients for an equivalent airgap g_e and secondary conductivity σ_e ,

$$g_e = g \frac{K_c K_{leak}}{K_m} > g \quad (20.34)$$

$$\sigma_e = \frac{\sigma_{Al}}{K_{skin} K_t} < \sigma_{Al} \quad (20.35)$$

We notice that all these effects contribute to a reduction in the realistic goodness factor G_i .

20.5 TRANSVERSE EDGE EFFECT IN SINGLE-SIDED LIM

For the single sided LIM with conductor sheet on iron secondary (Figure 20.15), a similar simplified theory has been developed where both the aluminum and saturated back iron contributions are considered [5,7].

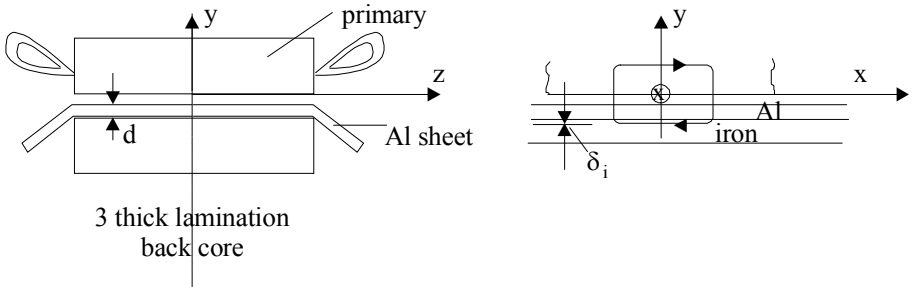


Figure 20.15 Single side LIM with 3 thick lamination secondary back core

The division of solid secondary back iron into three ($i = 3$) pieces along transverse direction leads to a reduction of eddy currents. This is due to an increase in the “transverse edge effect” in the back iron.

As there are no overhangs for secondary back iron ($c-a_e = 0$) the transverse edge effect coefficient K_t of (20.31) becomes

$$K_{ti} = \frac{1}{\tan \frac{\pi}{\tau} \frac{i}{a_e}} \quad (20.36)$$

$$1 - \frac{\frac{\pi}{\tau} \frac{i}{a_e}}{\frac{\pi}{\tau} \frac{i}{a_e}}$$

Now an equivalent iron conductivity σ_{ti} may be defined as

$$\sigma_{ti} = \frac{\sigma_{iron}}{K_{ti}} \quad (20.37)$$

The depth of field penetration in the secondary back iron δ_i is thus

$$\delta_i^{-1} = \text{Real} \left[\sqrt{\left(\frac{\pi}{\tau} \right)^2 + jS\omega_1 \mu_{iron} \frac{\sigma_{iron}}{K_{ti}}} \right] \quad (20.38)$$

The iron permeability μ_{iron} depends mainly on the tangential (along OX) flux density B_{xi} , in fact on its average over the penetration depth δ_i

$$B_{xi} = \frac{\tau}{\pi} \frac{B_g}{\delta_i} K_{pf}; \quad 1 \leq K_{pf} < 2 \quad (20.39)$$

where B_g is the given value of airgap flux density and K_{pf} accounts for the increase in back core maximum flux density in LIMs due to its open magnetic circuit along axis x [2, 7]; $K_{pf} = 1$ for rotary IMs.

Now we may define an equivalent conductivity σ_e of the aluminum to account for the secondary back iron contribution

$$\sigma_e = \frac{\sigma_{Al}}{K_{skinal}} \left[\frac{1}{K_{ta}} + \frac{\sigma_{iron} \delta_i K_{skinal}}{\sigma_{Al} K_{ti} d} \right] \quad (20.40)$$

K_{skinal} -is the skin effect coefficient for aluminum (20.3). K_{ta} -the transverse edge effect coefficient for aluminum.

In a similar way an equivalent airgap may be defined which accounts for the magnetic path in the secondary path by a coefficient K_p

$$g_e = (1 + K_p) g \frac{K_c K_{leak}}{K_{ma}} \quad (20.41)$$

$$K_p = \frac{\tau^2}{\pi^2} \frac{\mu_0}{2gK_c \delta_i \mu_{iron}} \quad (20.42)$$

K_p may not be neglected even if the secondary back iron is not heavily saturated.

Now the equivalent goodness factor G_e may be written as

$$G_e = \frac{\mu_0 \omega_1 \tau^2 \sigma_e d}{\pi^2 g_e} \quad (20.43)$$

The problem is that G_e depends on ω_1 , S , and μ_{iron} , for given machine geometry.

An iterative procedure is required to account for magnetic saturation in the back iron of secondary (μ_{iron}). The value of the resultant airgap flux density $B_g = \mu_0(H + H_0)$ may be obtained from (20.13) and (20.11) by neglecting the Z dependent terms

$$B_g \approx \frac{\mu_0 \tau}{\pi g_e} \frac{J_m}{(1 + jSG_e)}; \quad J_m = \frac{3\sqrt{2}W_l K_{wl} I_1}{p_1 \tau} \quad (20.44)$$

For given value of B_g , S , W_l , K_t , K_{ma} , K_{ti} are directly calculated. Then from (20.38)-(20.39) and the iron magnetization curve μ_{iron} (and B_{xi} , and δ_i) are iteratively computed.

Then σ_e and g_e are calculated from (20.40)-(20.42). Finally G_e , is determined. The primary phase current I_1 is computed from (20.44).

All above data serve to calculate the LIM thrust and other performance indices to be dealt with in the next paragraph in a technical longitudinal effect theory of LIMs.

20.6 A TECHNICAL THEORY OF LIM LONGITUDINAL END EFFECTS

Though we will consider the double sided LIM, the results to be obtained here are also valid for single-sided LIMs with same equivalent airgap g_e and secondary conductivity σ_e .

The technical theory as introduced here relies on a quasi-one dimensional model attached to the short (moving) primary. Also, for simplicity, the primary core is considered infinitely long but the primary winding is of finite length (Figure 20.16). All effects treated in previous paragraph enter the values of σ_e , g_e and G_e and the primary m.m.f. is replaced by the travelling current sheet $\underline{J}_1$ (20.2). Complex variables are used as sinusoidal time variations are considered.

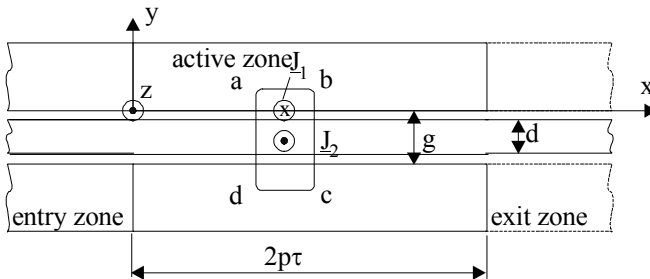


Figure 20.16 Double-sided LIM with infinitely long primary core

In the active zone ($0 \leq x \leq 2p_1\tau$) Ampere's law along abcd (Figure 20.16) yields

$$g_e \frac{\partial H_l}{\partial x} = J_1 e^{j\left(\omega_1 t - \frac{\pi}{\tau} x\right)} + J_2 d \quad (20.45)$$

H_t is the resultant magnetic field in airgap. It varies only along OY axis. Also the secondary current density J_2 has only one component (along OZ).

Faraday's law applied to moving bodies

$$\text{curl } E = -\frac{\partial B}{\partial x} + \bar{u} \times \bar{B}; \quad J = \sigma_e E \quad (20.46)$$

yields
$$\frac{\partial J_2}{\partial x} = j\omega_1 \mu_0 \sigma_e H_t + \mu_0 u \sigma_e \frac{\partial H_t}{\partial x} \quad (20.47)$$

where u is the relative speed between primary and secondary.

Equations (20.45) and (20.47) may be combined into

$$\frac{\partial^2 H_t}{\partial x^2} - \mu_0 u \sigma_e' \frac{\partial H_t}{\partial x} - j\omega_1 \mu_0 \sigma_e' H_t = -j \frac{\pi}{\tau g_e} J_m e^{-j \frac{\pi}{\tau} x} \quad (20.48)$$

$$\sigma_e' = \sigma_e \frac{d}{g_e} \quad (20.49)$$

The characteristic equation of (20.48) becomes

$$\gamma^2 - \mu_0 \sigma_e' u \gamma - j\omega_1 \mu_0 \sigma_e' = 0 \quad (20.50)$$

Its roots are
$$\gamma_{1,2} = \pm \frac{a_1}{2} \left[\sqrt{\frac{b_1 + 1}{2}} \pm 1 + j \sqrt{\frac{b_1 - 1}{2}} \right] = \gamma_{1,2r} \pm j\gamma_i \quad (20.51)$$

with

$$a_1 = \mu_0 \sigma_e' u = \frac{\pi}{\tau} G_e (1 - S) \quad (20.52)$$

$$b_1 = \sqrt{1 + \left(\frac{4}{G_e (1 - S)^2} \right)^2}$$

The complete solution of H_t within active zone is

$$H_{ta}(x) = \underline{A}_t e^{\gamma_1 x} + \underline{B}_t e^{\gamma_2 x} + \underline{B}_n e^{-j \frac{\pi}{\tau} x} \quad (20.54)$$

with
$$\underline{B}_n = \frac{j \tau J_m}{\pi g_e (1 + S G_e)}; \quad S = \frac{u_s - u}{u_s} \quad (20.55)$$

The coefficient $e^{j\omega_1 t}$ is understood.

In the entry ($x < 0$) and exit ($x > 2p_1\tau$) zones there are no primary currents. Consequently

$$H_{\text{entry}} = \underline{C}_e e^{\gamma_1 x}; \quad x \leq 0 \quad (20.56)$$

$$H_{\text{exit}} = \underline{D}_e e^{\gamma_2 x}; \quad x > 2p_1\tau \quad (20.57)$$

At $x = 0$ and $x = 2p_1\tau$, the magnetic fields and the current densities are continuous

$$(H_{\text{entry}})_{x=0} = (H_{\text{ta}})_{x=0} \quad (20.58)$$

$$(H_{\text{exit}})_{x=2p_1\tau} = (H_{\text{ta}})_{x=2p_1\tau} \quad (20.59)$$

$$\left(\frac{\partial H_{\text{entry}}}{\partial x} \right)_{x=0} = (J_2)_{x=0}; \quad \left(\frac{\partial H_{\text{exit}}}{\partial x} \right)_{x=2p_1\tau} = (J_2)_{x=2p_1\tau} \quad (20.60)$$

These conditions lead to

$$\underline{A}_t = -j \frac{J_m}{\underline{D}} \left(\gamma_2 \frac{\tau}{\pi} + SG_e \right) e^{-2p_1 \tau \gamma_1} \quad (20.61)$$

$$\underline{B}_t = j \frac{J_m}{\underline{D}} \left(\gamma_1 \frac{\tau}{\pi} + SG_e \right) \quad (20.62)$$

$$\underline{D} = g_e (\gamma_2 - \gamma_1) (1 + jSG_e) \quad (20.63)$$

$$G_e = \frac{\tau^2}{\pi^2} \omega_l \mu_0 \sigma_e \frac{d}{g_e} \quad (20.64)$$

20.7 LONGITUDINAL END-EFFECT WAVES AND CONSEQUENCES

The above field analysis enables us to investigate the dynamic longitudinal effects.

Equation (20.54) reveals the fact that the airgap field H_t and its flux density B_t has, besides the conventional unattenuated wave, two more components a forward and a backward travelling wave, because of longitudinal end effects. They are called end effect waves

$$\underline{B}_{\text{backward}} = -j\mu_0 \frac{J_m}{\underline{D}} \left(\gamma_2 \frac{\tau}{\pi} + SG_e \right) e^{(\gamma_{1r} + j\gamma_{1i})(x - 2p_1\tau)} \quad (20.65)$$

$$\underline{B}_{\text{forward}} = j\mu_0 \frac{J_m}{D} \left(\gamma_1 \frac{\tau}{\pi} + SG_e \right) e^{(\gamma_{2r} - j\gamma_i)x} \quad (20.66)$$

They may be called the exit and entry end-effect waves respectively. The real parts of $\gamma_{1,2}$ (γ_{1r}, γ_{2r}) determine the attenuation of end-effect waves along the direction of motion while the imaginary part $j\gamma_i$ determines the synchronous speed (u_{se}) of end effect waves

$$u_{se} = \frac{\omega_l}{\gamma_i}; \quad \tau_e = \frac{\pi}{\gamma_i} \quad (20.67)$$

The values of $1/\gamma_{1r}$ and $1/\gamma_{2r}$ may be called the depths of the end effect waves penetration in the (along) the active zone.

Apparently from (20.51)

$$\frac{1}{\gamma_{1r}} \ll \frac{1}{\gamma_{2r}}; \quad \frac{1}{\gamma_{1r}} < (8 \div 10) \cdot 10^{-3} \text{ m} \quad (20.68)$$

Consequently, the effect of backward (exit) end effect wave is negligible. Not so with the forward (entry) end effect wave which attenuates slowly in the airgap along the direction of motion.

The higher the value of goodness factor G_e and the lower the slip S , the more important the end effect waves are. High G_e means implicitly high synchronous speeds.

The pole pitch ratio of end-effect waves (τ_e/τ) is

$$\frac{\tau_e}{\tau} = \frac{2\sqrt{2}}{G_e(1-S) \left(\sqrt{1 + \left(\frac{4}{G_e(1-S)^2} \right)^2} - 1 \right)} \geq 1 \quad (20.69)$$

It may be shown that $\tau_e/\tau \geq 1$ and is approaching unity (at $S = 0$) for large goodness factor values G_e .

The conventional thrust F_{xc} is

$$F_{xc} = \mu_0 a_e J_m \operatorname{Re} \left(\int_0^{2p_1\tau} B_a^* dx \right) \quad (20.70)$$

The end-effect force F_{xe} has a similar expression

$$F_{xe} = \mu_0 a_e J_m \operatorname{Re} \left(\int_0^{2p_1\tau} B_t^* e^{\gamma_2^* x} e^{-j\frac{\pi}{\tau} x} dx \right) \quad (20.71)$$

The ratio f_e of these forces is a measure of end effect influence.

$$f_e = \frac{F_{xe}}{F_{xc}} = \frac{\operatorname{Re} \left[B_t^* \frac{-1 + \exp 2p_1 \tau \left(\gamma_2^* - j \frac{\pi}{\tau} \right)}{\left(\gamma_2^* - j \frac{\pi}{\tau} \right)} \right]}{\operatorname{Re} [B_n^* 2p_1 \tau]} \quad (20.72)$$

Or, finally,

$$f_e = \frac{1 + S^2 G_e^2}{S G_e} \operatorname{Re} \left\{ \frac{j \left(\frac{\gamma_1 \tau}{\pi} + S G_e \right) \left(\exp \left[2p_1 \tau \left(\gamma_2^* \frac{\tau}{\pi} - j \right) \right] - 1 \right)}{\frac{\tau}{\pi} (\gamma_2^* - \gamma_1^*) (1 - j S G_e) 2p_1 \pi \left(\gamma_2^* \frac{\tau}{\pi} - j \right)} \right\} \quad (20.73)$$

As seen from (20.73), f_e depends only on slip S , realistic goodness factor G_e , and on the number of poles $2p_1$.

Quite general p.u. values $(F_{xe})_{p.u.}$ of F_{xe} may be expressed as

$$(F_{xe})_{p.u.} = F_{xe} \frac{\pi^2 g_e}{a_e \mu_0 J_m^2} \quad (20.74)$$

$(F_{xe})_{p.u.}$ depends only on $2p_1$, S and G_e and is depicted in [Figure 20.17a, b, c](#).

The quite general results on [Figure 20.17](#) suggest that

- The end effect force at zero slip may be either propulsive-positive-(for low G_e values or (and) large number of poles) or it may be of braking character (negative)-for high G_e or (and) smaller number of poles.
- For a given number of poles and zero slip, there is a certain value of the realistic goodness factor G_{eo} , for which the end effect force is zero. This value of G_e is called the optimum goodness factor.
- For large values of G_e , the end effect force changes sign more than once as the slip varies from 1 to zero.
- The existence of the end effect force at zero slip is a distinct manifestation of longitudinal end effect.

Further on the airgap flux density $B_g = \mu_0 H_{ta}$ (see (20.54)) has a nonuniform distribution along OX that accentuates if S is low, goodness factor G_e is high and the number of poles is low.

Typical qualitative distributions are shown in [Figure 20.18](#).

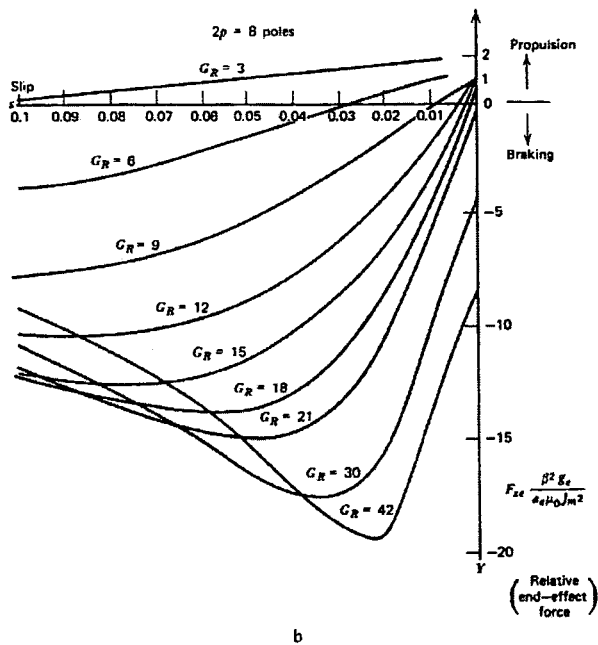
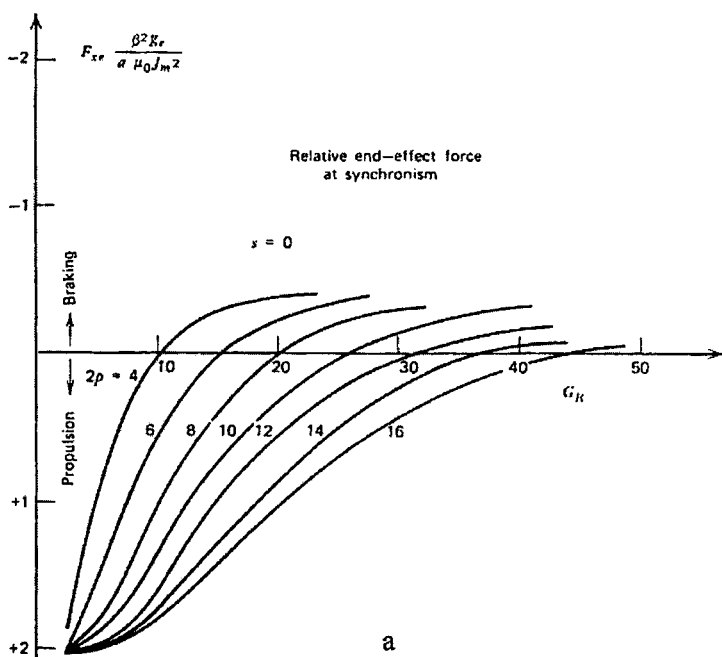


Figure 20.17 End effect force F_{se} in p.u.
a.) at zero slip; b.) at small slips; c.) versus normalized speed (continued)

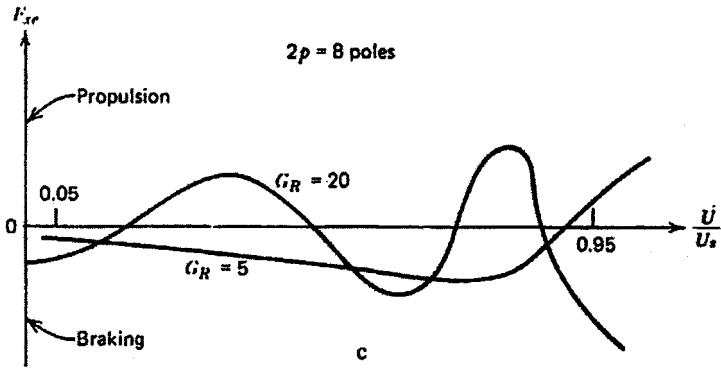


Figure 20.17 (continued)

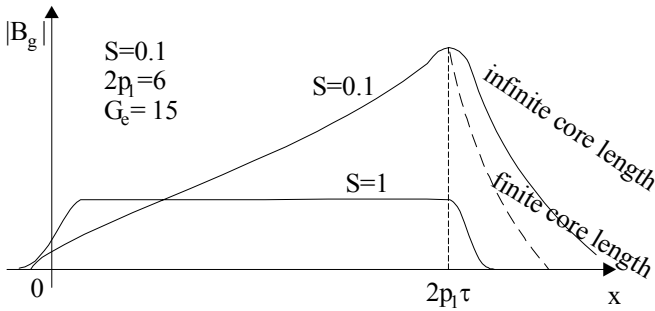


Figure 20.18 Airgap flux density distribution along OX

The problem is similar in single sided LIMs but there the saturation of secondary solid iron requires iterative computation procedures.

A nonuniform distribution shows also the secondary current density J_2

$$\underline{J}_2 = \frac{g_e}{d} \left[\gamma_1 \underline{A}_t e^{\gamma_1 x} + \gamma_2 \underline{B}_t e^{\gamma_2 x} - \frac{j S G_e \underline{J}_m e^{-j \frac{\pi}{\tau} x}}{g_e (1 + j S G_e)} \right] \quad (20.76)$$

Higher current density values are expected at the entry end ($x = 0$) and/or at the exit end at low values of slip for high goodness factor G_e and low number of poles $2p_1$ [5, pp. 271].

Consequently, the secondary plate losses are distributed nonuniformly along the direction of motion in the airgap [5, pp. 231].

Also, the propulsion force is not distributed uniformly along the core length. In presence of large longitudinal end effects, the thrust at entry end goes down to zero, even to negative values [5, pp.232].

Similar aspects occur in relation to normal forces in double-sided and single-sided LIMs [5, pp. 232].

20.8 SECONDARY POWER FACTOR AND EFFICIENCY

Based on the secondary current density $\underline{J}_2$ distribution (20.76), the power losses in the secondary P_2 are

$$P_2 = \frac{a_e d^2}{\sigma_e g_e} \int_{-\infty}^{\infty} (\underline{J}_2 \underline{J}_2^*) dx \quad (20.77)$$

Similarly, the reactive power Q_2 in the airgap is

$$Q_2 = a_e \omega_l \mu_0 g_e \int_{-\infty}^{\infty} (\underline{H}_{ta} \underline{H}_{ta}^*) dx \quad (20.78)$$

The secondary efficiency η_2 is

$$\eta_2 = \frac{uF_x}{uF_x + P_2}; \quad F_x = F_{xc} + F_{xe} \quad (20.79)$$

The secondary power factor $\cos \varphi_2$ is

$$\cos \varphi_2 = \frac{(uF_x + P_2)}{\sqrt{(uF_x + P_2)^2 + Q_2^2}} \quad (20.80)$$

Longitudinal end-effects deteriorate both the secondary efficiency and power factor. Typical numerical results for a super-high speed LIM are shown in [Figure 20.19](#).

So far, we considered the primary core as infinitely long. In reality, this is not the case. Consequently, the field in the exit zone decreases more rapidly ([Figure 20.18](#)) and thus the total secondary power losses are in fact, smaller than calculated above.

However, due to the same reason, at exit end there will be an additional reluctance small force [3, pp.74 - 79].

Numerical methods such as FEM would be suitable for a precise estimation of field distribution in the active, entry, and exit zones. However to account for transverse edge effect also, 3D FEM is mandatory.

Alternatively, 3D multilayer analytical methods have been applied successfully to single-sided LIMs with solid saturated and conducting secondary back iron [10 - 12] for reasonable computation time.

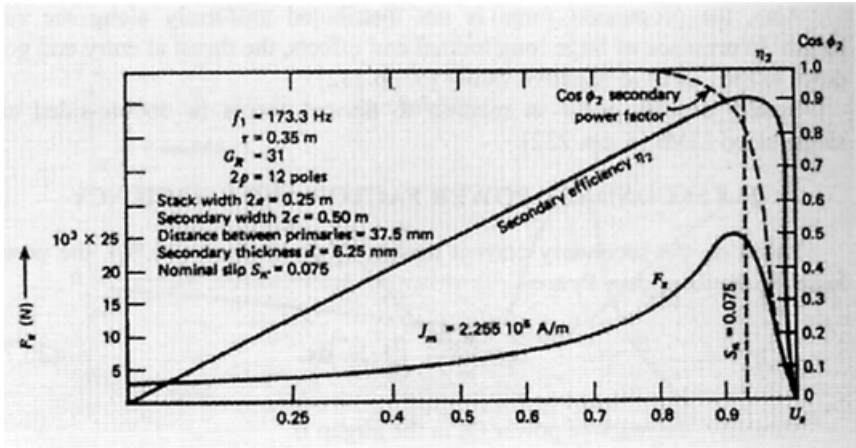


Figure 20.19 LIM secondary efficiency η_2 and power factor $\cos \phi_2$

20.9 THE OPTIMUM GOODNESS FACTOR

As we already noticed, the forward end effect wave has a longitudinal penetration depth of $\delta_{\text{end}} = 1/\gamma_{2r}$. We may assume that if

$$\frac{(\delta_{\text{end}})_{S=0}}{2p_1\tau} \leq 0.1 \quad (20.81)$$

the longitudinal end-effect consequences are negligible for $2p_1 \geq 4$. Condition (20.81) involves only the realistic goodness factor, G_o , and the number of poles.

Indirectly condition (20.81) is related to frequency, pole pitch (synchronous speed), secondary sheet thickness, conductivity, total airgap etc.

Consequently two LIMs of quite different speeds and powers may have the same longitudinal effect relative consequences if G_e and $2p_1$ and slip S are the same. [13]

End effect compensation schemes have been introduced early [2,3] but they did not prove to produce overall (global) advantages in comparison with well designed LIMs.

By well designed LIMs for high speed, we mean those designed for zero longitudinal end-effect force at zero slip. That is, designs at optimum goodness factor G_{eo} [5, pp. 238] which is solely dependent on the number of poles $2p_1$ (Figure 20.20).

G_{eo} is a rather intuitive compromise as higher G_e leads to both conventional performance enhancement and increase in the longitudinal effect adverse influence on performance.

LIMs where the dynamic longitudinal end effect may be neglected are called low speed LIMs or linear induction actuators while the rest of them are called high speed LIMs.

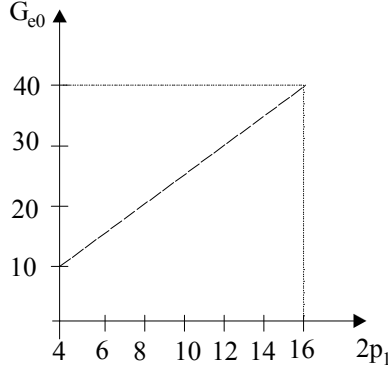


Figure 20.20 The optimum goodness factor

High speed LIMs are used for transportation-urban and inter-urban. In urban (suburban) transportation the speed seldom goes above 20(30) m/s but this is enough to make the longitudinal end effects worth considering, at least by global thrust correction coefficients. [14]

20.10 LINEAR FLAT INDUCTION ACTUATORS

Again, we mean by linear induction actuators (LIAs) low speed short travel, linear induction motor drives for which the dynamic longitudinal end effect may be neglected (20.81).

Most LIAs are single-sided (flat and tubular) with short primary and long conductor-sheet-iron or ladder secondary in a laminated slotted core. For double sided LIA, the long primary and short (moving) secondary configuration is of practical interest.

a. The equivalent circuit

All specific effects-airgap leakage, aluminum plate skin effect and transverse edge effects—have been considered and their effects lumped into equivalent airgap g_e (2.41) and aluminum sheet conductivity σ_e (2.40).

These expressions also account for the solid secondary back iron contribution in eddy currents and magnetic saturation. The secondary resistance R_2' reduced to the primary and the magnetizing reactance X_m (the secondary leakage reactance is neglected) can be adapted from (20.24)-(20.25) as

$$X_m = \frac{\mu_0 \omega_1 (W_1 K_{w1})^2 \tau (2a_e)}{\pi^2 p_1 g_e (S \omega_1, I_1)} = K_m W_1^2 \quad (20.82)$$

$$R_2' = \frac{X_m}{G_e} = \frac{12 (W_1 K_{w1})^2 a_e}{p_1 \tau d \sigma_e (S \omega_1, I_1)} = K_{R2} W_1^2 \quad (20.83)$$

So in fact both X_m and R_2' vary with slip frequency $S\omega_1$ and the stator current due to skin effect and transverse edge effect in both the conducting sheet and the back iron (if it is made of solid iron).

The equivalent circuit of rotary IM is now valid for LIAs, but with the variable parameters X_m and R_2' (Figure 20.21).

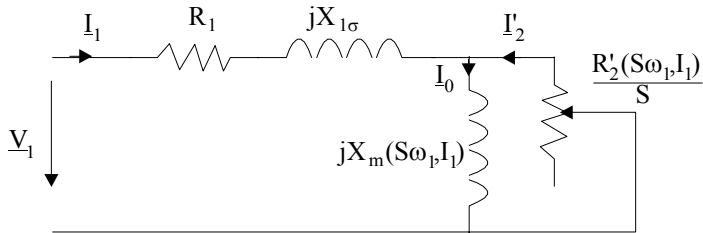


Figure 20.21 Linear induction actuator equivalent circuit

To fully exploit the equivalent circuit, we still need the expressions of primary resistance and leakage reactance R_1 and $X_{l\sigma}$.

$$R_1 = \frac{1}{\sigma_{Co}} \frac{(4a + 2l_{ec})}{W_1 I_1} W_1^2 j_{con}^2 = K_{R1} W_1^2 \quad (20.84)$$

where l_{ec} is the coil end connection length, j_{con} -rated current density, σ_{Co} -copper electrical conductivity.

$$X_{l\sigma} = \frac{2\mu_0\omega_l}{p_1 q} [(\lambda_s + \lambda_d)2a + \lambda_f l_{ec}] W_1^2 = K_{l\sigma} W_1^2 \quad (20.85)$$

$$\lambda_s \approx \frac{1}{12} \frac{h_s}{b_s} (1 + 3\beta'); \quad \beta' = \frac{y}{\tau} \text{ (coil span)} \quad (20.86)$$

$$\lambda_d = \frac{5 \frac{g}{b_s}}{5 + 4 \frac{g}{b_s}}; \quad b_s, h_s - \text{open slot width and height} \quad (20.87)$$

$$\lambda_f \approx 0.3q(3\beta - 1); \quad l_{ec} \approx K_f \tau; \quad K_f = 1.3 \div 1.6 \quad (20.88)$$

$\lambda_s, \lambda_d, \lambda_f$ -slot, differential, end connection specific geometrical permeances.

The primary phase m.m.f. $W_1 I_1$ (RMS) is

$$W_1 I_1 = j_{con} \left(\frac{\tau}{3q} \right)^2 K_d^2 \frac{h_s}{b_s} K_{fill} \quad (20.89)$$

where $K_d = \frac{3qb_s}{\tau}$, the open slot width/slot pitch, is 0.5 to 0.7 for LIAs; h_s/b_s is the slot aspect ratio and varies between 3 and 6(7) for LIAs; K_{fill} is the slot fill factor and varies in the interval $K_{fill} = 0.35-0.45$ for round wire random wound coils and is 0.6-0.7 for preformed rectangular wire coils.

b. Performance computation

Making use of the equivalent circuit (Figure 20.21), with all parameters already calculated, we may simply determine the thrust F_x ,

$$F_x = \frac{3I_2'^2 R_2'}{S \cdot 2\tau f_1} = \frac{3I_1'^2 R_2'}{S \cdot 2\tau f_1 \left[\left(\frac{1}{SG_e} \right)^2 + 1 \right]} \quad (20.90)$$

The efficiency η_1 and power factor $\cos \phi_1$ are

$$\eta_1 = \frac{2\tau f_1 (1-S) F_x}{2\tau f_1 F_x + 3I_1'^2 R_1} \quad (20.91)$$

$$\cos \phi_1 = \frac{2\tau f_1 F_x + 3I_1'^2 R_1}{3V_{1f} I_1} \quad (20.92)$$

As expected, the realistic goodness factor G_e plays an important role in thrust production (for given stator current I_1) and in performance (Figure 20.22).

The peak thrust for given stator current follows from (20.90) with

$$\frac{\partial F_x}{\partial S} = 0 \rightarrow S_k G_e = \pm 1; F_{xk} = \frac{\pm 3I_1'^2 R_2' G_e}{2\tau f_1 \cdot 2} \quad (20.93)$$

Evidently (20.93) is valid only if R_2' , X_m , G_e are constant.

In many low speed applications, the realistic goodness is around unity. If peak thrust/ current is needed at standstill, then $S_k = 1$ and thus the design goodness factor at start is

$$G_e = \omega_1 \sigma_e \mu_0 \frac{d}{g_e} \frac{\tau^2}{\pi^2} = 1 \quad \text{for } S_k = 1 \quad (20.94)$$

Here $\omega_1 = 2\pi f_1$ is the primary frequency at start. If a variable frequency converter is used, the starting frequency is reduced.

The thrust/secondary losses is, from 20.90,

$$\frac{F_x}{P_2} = \frac{F_x}{3I_2'^2 R_2'} = \frac{1}{2\tau f_1 \cdot S} \quad (20.95)$$

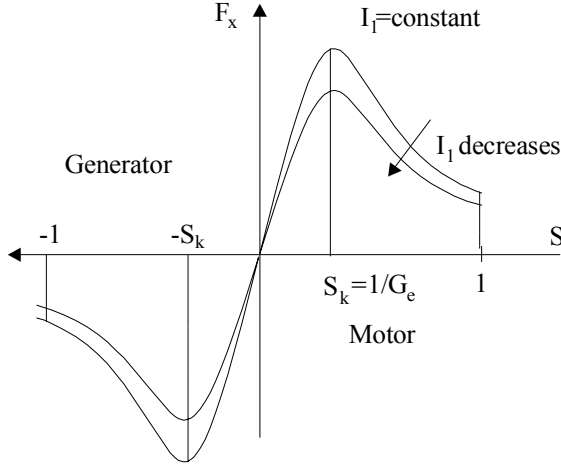


Figure 20.22 Thrust versus slip

For $S = 1$ the smaller the frequency f_1 at start, for given τ , the larger the thrust/ secondary conductor losses.

However as the value of τ decreases, to reduce the back core height, for given airgap flux density, the starting frequency does not fall below (5-6)Hz for most practical cases.

c. Normal force in single-sided configurations

The normal force between single-sided LIAs primary and secondary has two components

- An attraction force F_{na} between the primary core and the secondary back iron produced by the normal airgap flux density component.
 - A repulsion force F_{nr} between the primary and secondary current m.m.f.s.
- As the longitudinal end effect is absent, F_{na} is

$$F_{na} = \frac{2a_e |\underline{B}_g|^2 2p_1 \tau}{2\mu_0} \quad (20.96)$$

Making use of (20.44) for $\underline{B}_g$, (20.96) becomes

$$F_{na} \approx \frac{a_e \mu_0 J_{lm}^2 \tau^2 2p_1 \tau}{\pi^2 g_e^2 (1 + S^2 G_e^2)} \quad (20.97)$$

To calculate the repulsive force F_{nr} , the tangential component (along OX) of airgap magnetic field H_x is needed. The value of H_x along the primary surface is

$$H_x = J_m; \quad B_x = \mu_0 J_m \quad (20.98)$$

$$F_{nr} = 4 \frac{a_e}{2} \tau p_1 d \mu_0 \operatorname{Re}[\underline{J}_2^* \underline{J}_m] \quad (20.99)$$

But
$$\underline{J}_2 = S \omega_1 \sigma_e \frac{\pi}{\tau} \underline{B}_g \quad (20.100)$$

Finally,
$$F_{nr} \approx -2a_e \tau S \omega_1 a_e d p_1 \frac{\tau^2}{\pi^2} \frac{(\mu_0 J_m)^2 S G_e}{g_e (1 + S^2 G_e^2)} \quad (20.101)$$

The net normal force F_n is

$$F_n = F_{na} + F_{nr} = 2a_e p_1 \tau \frac{\mu_0 J_m^2 \tau^2}{\pi^2 g_e^2 (1 + S^2 G_e^2)} \left[1 - \left(\frac{g_e \pi}{\tau} \right)^2 S^2 G_e^2 \right] \quad (20.102)$$

From (20.102), the net normal force becomes repulsive if

$$S G_e > \frac{\tau}{g_e \pi} \quad (20.103)$$

which is not the case in most low speed LIMs (LIAs) even at zero speed ($S = 1$).

Core losses in the primary core have been neglected so far but they may be added as in rotary IMs. Anyway, they tend to be relatively smaller as the airgap flux density is only $B_{gn} = (0.2-0.45)T$, because of the rather large magnetic airgap (air plus conductor sheet thickness).

d. A numerical example

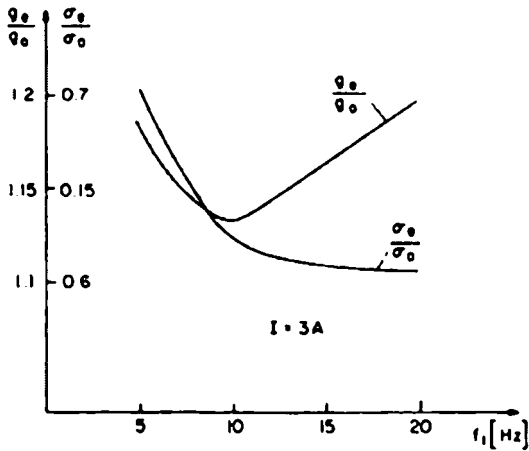


Figure 20.23 Equivalent airgap (g_e/g_0) and secondary conductivity (σ_e/σ_{Al}) thrust F_x and goodness factor G_e versus frequency at standstill (continued)

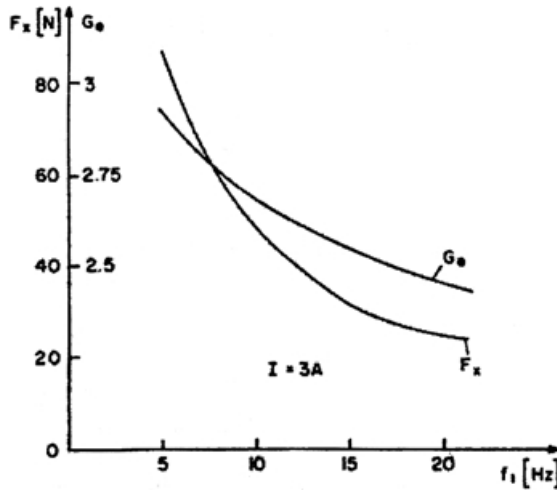


Figure 20.23 (continued)

Let us consider a sheet on iron secondary (single sided) LIA. The initial data are pole pitch $\tau = 0.084$ m, $2a_e = 0.08$ m, the number of secondary back iron “solid” laminations is $i = 3$, number of poles $2p_1 = 6$, $q = 2$, $W_1 = 480$ turns/phase, $g = 0.012$ m (out of which $d_{Al} = 0.006$ m), aluminum plate width $2c = 0.12$ m, $\sigma_{Al} = 3.5 \cdot 10^7$ (Ωm)⁻¹, $\sigma_{iron} = 3.55 \cdot 10^6$ (Ωm)⁻¹, slot depth $h_s = 0.045$ m, slot width $b_s = 0.009$ m, $\beta' = y/\tau = 5/6$ (coil span ratio).

The equivalent conductivity and airgap ratios versus frequency at start ($S = 1$) and the thrust and goodness factor for given (constant) stator current $I_1 = 3$ A (RMS) are shown in Figure 20.23.

Figure 20.23 summarizes the influence of skin effect, saturation, and eddy currents in the secondary back iron.

As the frequency increases G_e slowly deriorates (decreases) because the skin depth δ_i in iron decreases and so the back iron contribution to the equivalent airgap increases.

The airgap ratio shows a minimum. It is the σ_e/g_e ratio that counts in the goodness factor. For our case, approaching $f_1 = 50\text{Hz}$ at start seems good choice.

e. Design methodology by example

In a single sided LIA, the back iron in the secondary is necessary to provide a low magnetic reluctance path for the magnetic flux which crosses the airgap. Usage of a solid back iron is imposed by economic reasons. If a laminated secondary core is to be used to step-up performance, then a ladder secondary in slots seems the adequate choice. This case will be investigated in a separate paragraph.

For the time being, we consider the case of secondary solid back iron. Due to larger equivalent airgap per pole pitch ratio (g_e/τ) both, the efficiency and power factor are not particularly high, they do not seem the best design criteria to start with.

We also need some data from past experience mechanical gap $g-d = 1$ to 4 mm, airgap flux density $B_g = 0.15$ to 0.35T, aluminum sheet thickness $d = 2$ to 4 mm, slot depth/slot width $K_{slot} = h_s/b_s = 3$ to 6.

The number of poles should be $2p_1 = 4, 6, 8$. The case of $2p_1 = 2$ should be used only in very small thrust applications. The secondary overhangs ($c-a$) should not surpass τ/π . Let us consider $c-a = \tau/\pi$.

The basic design criterion is chosen as maximum thrust/given stator current (20.93) which yields

$$S_k G_e = 1 \quad (20.104)$$

This case corresponds approximately to maximum thrust per conductor losses.

Design specifications are considered here to be

- Peak thrust up to rated speed $F_{xk} = 1500N$
- Rated speed $u_n = 6m/s$
- Rated frequency $f_{1n} = 50Hz$
- Rated phase voltage $V_1 = 220V$ (RMS) star connection
- Variable frequency converter supply
- Excursion length $L_1 = 50m$

The design algorithm unfolds as follows.

The peak thrust F_{xk} (from 20.90 with R'_2 from (20.83)) is

$$F_{xn} = (F_{xk})_{S_k G_e = 1} = \frac{3I_1^2 (W_l K_{wl})^2 12a_e}{d_{Al} \tau p_1 \sigma_e (2\tau f_1 - u_n)} \frac{1}{2} = 1500N \quad (20.105)$$

Also from (20.104) with G_e from (20.43)

$$S_k G_e = 1 = 2\mu_0 \frac{\tau^2 f_1}{\pi g_e} S_k d_a \sigma_e \quad (20.106)$$

The rated speed

$$u_n = 2f_1 \tau (1 - S_k) \quad (20.107)$$

Also from (20.43) the airgap flux density for $S_k G_e = 1$ is

$$B_{gn} = (B_g)_{S_k G_e = 1} = \frac{3\sqrt{2} W_l K_{wl} \mu_0 I_1}{g_e p_1 \pi \sqrt{2}}; \quad B_{gn} = 0.3T \quad (20.108)$$

The secondary back iron tangential flux density B_{xin} should also be limited to a given value, say $B_{xin} = 1.7T$ (even higher values may be accepted). From (20.39) the secondary iron penetration depth required δ_{in} is

$$\delta_{in} = \frac{\tau B_{gn} K_{pf}}{\pi B_{xin}}; \quad K_{pf} = 1.6 \quad (20.109)$$

We may also choose the rated current density $j_{con} = 4A/mm^2$ (for no cooling), the number of slots per pole and phase $q = 2$, slot aspect ratio $K_{slot} = (3-6)$ and slot/slot pitch ratio $K_d = 3qb_s/\tau = 0.5-0.7$, slot fill factor $K_{fill} = 0.35-0.45$. With these data the phase m.m.f. $W_1 I_{1n}$ is

$$W_1 I_{1n} = C_s p_1 \tau^2; \quad C_s = \frac{1}{9q} K_d^2 K_{slot} K_{fill} j_{con} \quad (20.110)$$

$$\text{with } K_d = \frac{3b_s q}{\tau} = 1 - \frac{B_{gn}}{B_{tn}}; \quad B_{tn} - \text{primary tooth design flux density} \quad (20.111)$$

The unknowns of the problem as defined so far are τ , a_e/τ , p_1 , $S_k \omega_1$ and d_{Al} and only an iterative procedure can lead to solutions.

However, by choosing a_e/τ as a parameter the problem simplifies considerably.

From (20.109) and (20.38), we get

$$\tau^2 S_k \omega_1 = \frac{K_{ti} \pi^2}{\mu_0 \sigma_i} \sqrt{\left(\frac{B_{xin}}{B_{gn}} \right)^4 - 4} = c_\tau \quad (20.112)$$

The thrust F_{xk} , from (20.105) with (20.108) and (20.106), is

$$F_{xn} = 6C_s B_{gn} K_{w1} p_1 \tau^3 \left(\frac{a_e}{\tau} \right) \quad (20.113)$$

As the primary frequency f_1 is given we may choose also the number of poles as a parameter $2p_1 = 4, 6, \dots$. In this case from (20.113), the pole pitch τ may be calculated as

$$\tau = \sqrt[3]{\frac{F_{xn}}{6C_s B_{gn} K_{w1} p_1 \left(\frac{a_e}{\tau} \right)}} \quad (20.114)$$

Now $S_k \omega_1$ is

$$S_k \omega_1 = 2\pi f_1 - u_n \frac{\pi}{\tau} \quad (20.115)$$

Alternatively, we may have used (20.112) to find $S_k \omega_1$ in which case even f_1 from (20.115) could have been calculated. This would be the case with f_{1n} not specified. Anyway (20.112) is to be checked.

Now from (20.105-20.108)

$$\sigma_e d_{Al} = \frac{3C_s \tau \pi}{S_k \omega_l B_{gn}} \quad (20.116)$$

From (20.40) the value $\sigma_{Alskin} d_a$ is

$$\sigma_{Alskin} d_{Al} = \left(\frac{3C_s \tau \pi}{S_k \omega_l B_g} - \frac{\sigma_i \delta_i}{K_{ti}} \right) K_{ta} \quad (20.117)$$

As K_{ta} comes from Equations (20.14), (20.26-20.30), where G_e (which contains $\sigma_{Alskin} d_a$) is included, the computation of $\sigma_{Alskin} d_{Al}$ may be done only iteratively from these equations.

With $\sigma_{Alskin} d_{Al}$ known, again iteratively, d_{Al} may be determined (the skin effect depends on d_{Al}). A valid solution is found if $d_{Al} < g_e$.

With the above procedure the equivalent parameters entering the equivalent circuit, X_m , R_2' , R_1 , $X_{l\sigma}$ from (20.92-20.88), for rated speed u_n and thrust F_{xn} , may be calculated. The only unknown is now the number of turns per phase W_1

$$W_1 = \frac{V_1}{W_1 I_{ln} \left[\left(K_{R1} + \frac{K_{R2'}}{S_k (1 + (1/S_k G_e)^2)} \right)^2 + \left(K_{x1\sigma} + \frac{K_m}{(1 + (S_k G_e)^2)} \right)^2 \right]} \quad (20.118)$$

with $W_1 I_{ln}$ from (20.110), (20.118) provides a unique value for W_1 .

To choose the adequate a_e/τ ratio, which will lead to the best design, the cost C_t of both primary (C_p) and secondary (C_{sec}) are evaluated

$$C_t = C_p + C_{sec} \quad (20.119)$$

$$C_p = \frac{6W_1 I_{ln}}{J_{con}} (2a + K_1 \tau) \gamma_{Co} C_{Co} + \left[2p_1 \tau (1 - K_d) \cdot 2a K_{slot} \frac{K_d \tau}{3q} + 3\delta_i 2p_1 \tau \cdot 2a \right] \gamma_i C_{ii} \quad (20.120)$$

where γ_{Co} , γ_i , C_{Co} , C_{ii} are the specific weights and specific prices of copper and core respectively

$$C_{sec} = \left[\left(2a + \frac{2\tau}{\pi} \right) d_{Al} \gamma_{Al} C_{Al} + 3\delta_i 2a \gamma_i C_i \right] L_1 \quad (20.121)$$

γ_{Al} , C_{Al} are aluminum specific weight and specific cost, respectively, and L_1 is the total LIA secondary length (excursion length + primary length).

When $L_1 \gg 2p_1 \tau$, the secondary cost becomes an important cost factor. Thus a high a_e/τ ratio leads to a smaller length primary but also to a wider (large cost) secondary. The final choice is left to the designer. The apparent power S_1 may be used as an indicator of converter costs.

For our numerical case, the results in Figure 20.24 are obtained. The total cost/iron specific cost (C_t/C_{ii}) has a minimum for $a_e/\tau = 0.7$ but this corresponds

to a rather high S_1 (KVA). A high S_1 means a higher cost frequency converter. A good compromise would be $a_e/\tau = 0.9$, $S_k\omega_1 = 93\text{rad}\cdot\text{sec}$, $\tau = 0.085\text{m}$, $d_{Al} = 1.8\cdot 10^{-3}\text{m}$, $W_1 = 384\text{turns/phase}$, $I_{1n} = 32.757\text{A}$, $S_1 = 21.92\text{KVA}$.

Now that the LIAs dimensions are all know, the performance may be calculated iteratively for every (V_1, f_1) pair and speed U .

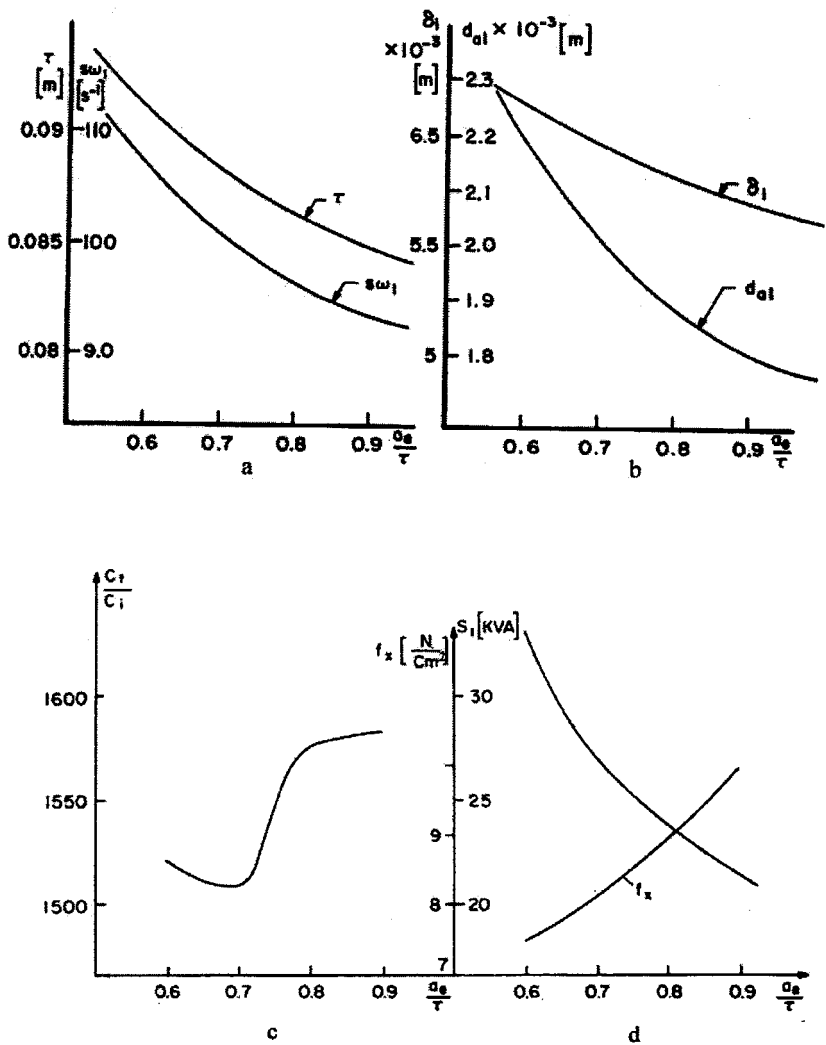


Figure 20.24 Design numerical results
a.) $S_k\omega_1$, b.) iron penetration depth δ_i aluminum thickness d_{Al} ,
c.) cost versus a_e/τ d.) S_1 (KVA), specific thrust $f_x(\text{N}/\text{cm}^2)$

For a double-sided LIA with aluminum sheet secondary the same design procedure may be used, after its drastic simplification due to the absence of solid iron in the secondary. Also the two-sided windings may be lumped into an equivalent one.

f. The ladder secondary

Lowering the airgap to $g = 1\text{mm}$ or even less in special applications with LIAs may be feasible. In such cases the primary slots should be semiclosed to reduce Carter's coefficient value and additional losses. The secondary may then be built from a laminated core with, again, semiclosed slots, for same reasons as above (Figure 20.25).

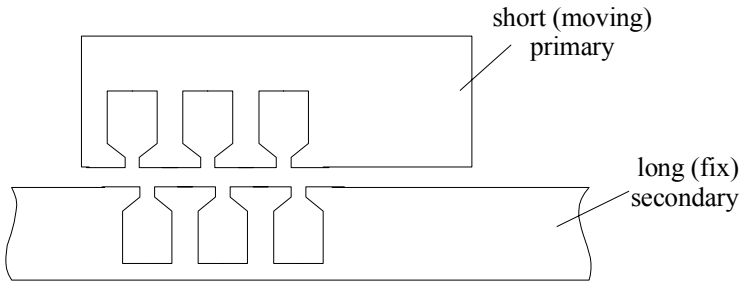


Figure 20.25 LIA with ladder secondary

To calculate the performance, the conventional equivalent circuit is used. The secondary parameter expressions are identical to those derived for the rotary IM. The adequate number of combinations of primary/secondary slots (per primary length) are the same as for rotary IM. In fact, the whole design process is the same as for the rotary IM.

The only difference is that the electromagnetic power is

$$P_{elm} = F_x 2\tau f_1 = \frac{3R_2' I_2'^2}{S} \quad (20.122)$$

instead of

$$P_{elm} = T_e \frac{\omega_1}{p_1} = \frac{3R_2' I_2'^2}{S} \quad (20.123)$$

for rotary IMs.

For short excursion (less than a few meters) and $g \approx 1\text{mm}$ quite good performance may be obtained with ladder secondary

20.11 TUBULAR LIAs

There are two main tubular LIA configurations one with longitudinal primary laminations stack and the other with ring shape laminations and secondary ring-shape conductors in slots (Figure 20.26).

Due to manufacturing advantages we consider here-except for liquid pump applications-disk shape laminations (Figure 20.26).

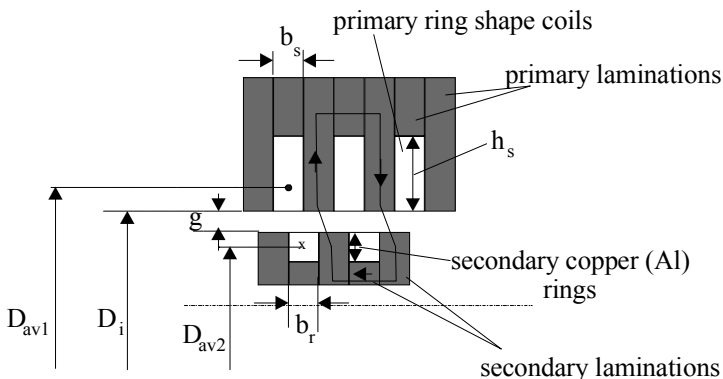


Figure 20.26 Tubular LIM with secondary conductor rings

The main problem of this configuration is that it has open slots on both sides of the airgap.

To limit the airgap flux harmonics (with all their consequences in “parasitic” losses and forces), for an airgap g ($g \approx 1\text{mm}$) the primary slot width b_s should not be larger than $(4-6)g$. Fortunately, this suffices for many practical cases.

The secondary ring width b_r should not be larger than $(3-4)g$. Another secondary effect originates in the fact that the back core flux path goes perpendicular to laminations.

Consequently, the total airgap is increased at least by the insulation total thickness along say $\approx 1/3\tau$. Fortunately, this increase by, g_{ad} , is not very large

$$g_{ad} = 0.03 \frac{1}{3} \tau \approx 0.01\tau \quad (20.124)$$

For a pole pitch $\tau = 30\text{mm}$ $g_{ad} = 0.3-0.4\text{mm}$, which is acceptable. A very good stacking factor of 0.97 has been assumed in (20.124). Also eddy current core losses will be increased notably in the back iron due to the perpendicular field (Figure 20.27).

Performing slits in the back-core disk shape laminations during stamping leads to a notable reduction of eddy current losses. Finally, the secondary is to be coated with mechanically resilient, nonconducting, nonmagnetic material for allowing the use of linear bearings.

Once these peculiarities are taken care of, the tubular LIM performance computation runs smoothly. The absence of end connections, both in the primary and secondary coils, is a definite advantage of tubular configurations.

So no transverse edge effect occurs in the secondary. The primary back iron is not likely to saturate but the secondary back iron may do so as its area for the half-pole flux is much smaller than in the primary.

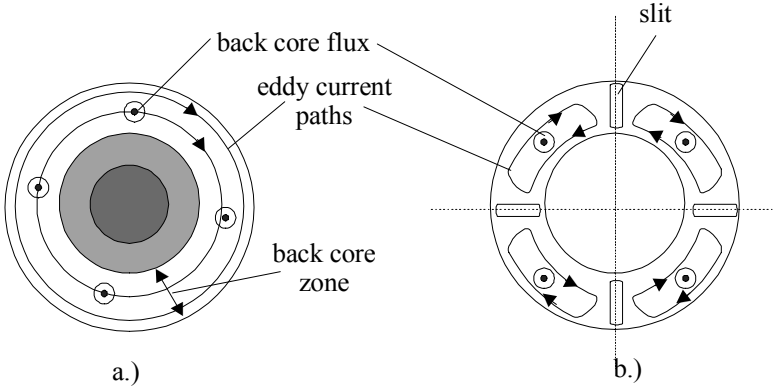


Figure 20.27 Tubular LIA back core disk-shape lamination a.) and its slitting b.) to reduce eddy current losses

The equivalent circuit parameters are

$$R_{lt} = \frac{1}{\sigma_{Co}} \frac{\pi D_{av1} J_{con}}{W_l I_{ln}} W_l^2 \quad (20.125)$$

$$X_{lt} = 2\mu_0 \omega_l \frac{\pi D_{av1}}{p_l q} (\lambda_{sl} + \lambda_{dl}) W_l^2 \quad (20.126)$$

We have just “marked” in (20.125-20.126) that the primary coil average length is πD_{av1} . For the secondary with copper rings

$$X_{2r}' = 2\mu_0 \omega_l \pi D_{av2} \cdot 12 \frac{(W_l K_{wl})^2}{N_{s2}} (\lambda_{s2} + \lambda_{d2}) \quad (20.127)$$

$$R_{2r}' = \frac{1}{\sigma_{Co}} \frac{\pi D_{av2}}{A_{ring}} \cdot 12 \frac{(W_l K_{wl})^2}{N_{s2}} \quad (20.128)$$

λ_s and λ_d are defined in (20.86)-(20.87), N_{s2} is the number of secondary slots along primary length, A_{ring} -copper ring cross-section.

The magnetisation reactance X_m is

$$X_m = \frac{6\mu_0 \omega_l}{\pi^2} (W_l K_{wl})^2 \frac{\pi D_i \tau}{p_l g K_c \left(1 + \frac{g_{ad}}{g} + K_{sat} \right)} \quad (20.129)$$

K_{sat} -iron saturation coefficient should be kept below 0.4-0.5.

For an aluminum sheet-secondary on laminated core

$$R_2' = \frac{6\pi(D_i - 2g - d_{Al})(W_l K_{wl})^2}{p_l \tau d_{Al} \sigma_{Al}}; \quad X_2' = 0 \quad (20.130)$$

The equivalent circuit (Figure 20.28) now contains also the secondary leakage reactance which is however small as the slots are open and not very deep.

Based on the equivalent circuit, all performance may be obtained at ease.

The tubular LIA is used for short stroke applications which implicitly leads to maximum speeds below 2-3m/s. Consequently, the design of tubular LIAs, in terms of energy conversion, should aim to produce the maximum thrust at standstill per unit of apparent power if possible.

The primary frequency may be varied (and controlled) through a static power converter, and chosen below the industrial one.

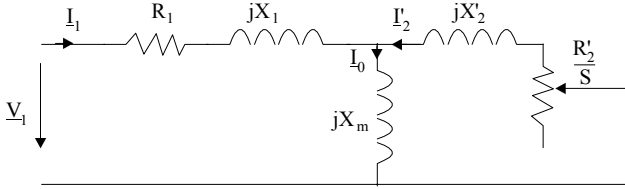


Figure 20.28 Equivalent circuit of tubular LIA with secondary copper rings in slots

In terms of costs both the LIA and power converter costs are to be considered. Hereby, however, we illustrate the criterion of maximum thrust at standstill.

The electromagnetic power P_{elm} is

$$P_{elm} = F_x 2\tau f_l = 3R_e I_1^2 \quad (20.131)$$

$$R_e = \frac{\frac{R_2'}{S} X_m^2}{\left(\frac{R_2'}{S}\right)^2 + (X_m + X_2')^2} \quad (20.132)$$

$$\text{or with } G_e = \frac{X_m}{R_2'} \quad R_e = \frac{R_2'}{S \left[\left(\frac{1}{SG_e} \right)^2 + \left(1 + \frac{X_2'}{X_m} \right)^2 \right]} \quad (20.133)$$

Now the maximum thrust is obtained for

$$(S_k G_e)_{opt} = \frac{1}{1 + \frac{X_2'}{X_m}} \quad (20.134)$$

For standstill, $S_k = 1$. When initiating the design process, the ratio X_2'/X_m is not known and can only be assigned a value to be adjusted iteratively later on. A good start would be $X_2'/X_m = 0.2$.

From now on, the design methodology developed for the flat LIA may be used here.

a. A numerical example

Let us consider a tubular LIA with copper ring secondary and the data primary bore diameter $D_i = 0.15$ m, primary external diameter $D_e = 0.27$ m, pole pitch $\tau = 0.06$ m, $q = 1$ slots/pole/phase, primary slot depth $h_s = 0.04$ m, primary slot width $b_s = 0.0125$ m, number of poles $2p_1 = 8$, the secondary slot pitch $\tau_{s2} = 10^{-2}$ m, secondary slot width $b_{s2} = 6 \cdot 10^{-3}$ m, secondary slot depth $h_{s2} = 4 \cdot 10^{-3}$ m, airgap $g = 2 \cdot 10^{-3}$ m, current density $j_{con} = 3 \cdot 10^6$ A/m², slot fill factor $K_{fill} = 0.6$.

Let us determine the frequency f_2 for maximum thrust at standstill and the corresponding thrust, apparent power and conductor losses.

First, with the available data, we may calculate from (20.127) the secondary leakage reactance as a function of $f_1 W_1 K_{w1}$

$$X_2' = 4\mu_0 \pi D_{av2} \cdot \frac{12}{N_{s2}} (\lambda_{s2} + \lambda_{d2}) f_1 W_1^2 K_{w1}^2 = 1.97 \cdot 10^{-6} f_1 W_1^2 K_{w1}^2 \quad (20.135)$$

$$\text{with} \quad \lambda_{s2} = \frac{h_{s2}}{3b_{s2}} = \frac{4}{3 \cdot 6} = 0.33; \quad \lambda_{d2} = \frac{\frac{5g}{b_{s2}}}{5 + \frac{4g}{b_{s2}}} = \frac{\frac{5 \cdot 3}{6}}{5 + \frac{4 \cdot 3}{6}} = 0.57 \quad (20.136)$$

The secondary resistance (20.128) is

$$R_2' = \frac{1}{\sigma_{Co}} \frac{\pi D_{av2}}{h_{s2} b_{s2}} \cdot 12 (W_1 K_{w1})^2 = 0.876 \cdot 10^{-4} (W_1 K_{w1})^2 \quad (20.137)$$

The magnetic reactance X_m (20.129) becomes

$$X_m = \frac{6\mu_0 \omega_1}{\pi^2} (W_1 K_{w1})^2 \frac{\pi D_i \tau}{p_1 g \left(1 + \frac{g_{ad}}{g} + K_{sat} \right)} = 10^{-5} f_1 (W_1 K_{w1})^2 \quad (20.138)$$

Let assign $\frac{g_{ad}}{g} + K_{sat} = 0.5$.

Now the optimum goodness factor is (from (20.134))

$$G_0 = \frac{X_m}{R_2'} = 0.114 f_{ls} = \frac{1}{1 + \frac{X_2'}{R_2'}} = \frac{1}{1 + 2.24 \cdot 10^{-2} f_{ls}} \quad (20.139)$$

From (20.139), the primary frequency at standstill f_{1s} is $f_{1s} \approx 7.5\text{Hz}$. The primary phase m.m.f. $W_1 I_{1n}$ is

$$W_1 I_{1n} = p q n_s I_{1n} = p_1 q (h_s b_s K_{\text{fill}} J_{\text{con}}) = 4 \cdot 4 \cdot 10^{-2} \cdot 1.2 \cdot 10^{-2} \cdot 0.6 \cdot 3 \cdot 10^{-6} = 3456 \text{At} \quad (20.140)$$

Hence, the primary phase resistance and leakage reactance R_{1t} , X_{1t} are (20.125-20.126)

$$R_{1t} = \frac{1}{\sigma_{\text{Co}}} \frac{\pi D_{\text{avl}} J_{\text{con}}}{W_1 I_{1n}} W_1^2 = 1.035 \cdot 10^{-5} W_1^2 \quad (20.141)$$

with $D_{\text{avl}} = D_i + h_s = 0.15 + 0.04 = 0.19 \text{ m}$.

$$X_{1t} = 2\mu_0 \omega_1 \frac{\pi D_{\text{avl}}}{p_1 q} (\lambda_{s1} + \lambda_{d1}) W_1^2 = 2.483 \cdot 10^{-5} W_1^2 \quad (20.142)$$

The number of turns W_1 may be determined once the rated current I_{1n} is assigned a value. For $I_{1n} = 20\text{A}$, from (20.140), $W_1 = 3456/20 \approx 172$ turns/phase.

The apparent power S_1 does not depend on voltage (or on W_1) and is

$$(S_1)_{S=1} = 3I_1^2 \left| R_{1t} + jX_{1t} + \frac{jX_m(R_2' + jX_2')}{j(X_m + X_2') + R_2'} \right| \quad (20.143)$$

with $W_1 I_{1n}$ and R_{1t} , X_{1t} , X_m , R_2' , X_2' proportional to W_1^2 and with $K_{w1} = 1.00$ ($q = 1$) we obtain

$$S_1 \approx 3000 \text{VA}; \quad \cos \phi_1 \approx 0.52 \quad (20.144)$$

The corresponding phase voltage at start is

$$V_1 = \frac{S_1}{3I_{1n}} = \frac{3000}{3 \cdot 20} = 50.00 \text{V(RMS)} \quad (20.145)$$

The rated input power $P_1 = S_1 \cos \phi_1 = 3000 \cdot 0.52 = 1560 \text{W}$.

The primary conductor losses p_{cos} are

$$p_{\text{cos}} = 3R_{1t} I_{1n}^2 = 3 \cdot 0.135 \cdot 10^{-4} (3456)^2 = 370.86 \text{W} \quad (20.146)$$

The electromagnetic power P_{elm} writes

$$P_{\text{elm}} = P_1 - p_{\text{cos}} = 1560 - 370.86 = 1189 \text{W} \quad (20.147)$$

The thrust at standstill F_{xk} is

$$F_{\text{xk}} = \frac{P_{\text{elm}}}{2\tau f_1} = \frac{1189}{2 \cdot 0.06 \cdot 7.5} = 1321.26 \text{N} \quad (20.147')$$

Thus the peak thrust at standstill per watt is

$$f_{xw} = \frac{F_{xk}}{P_1} = \frac{1321.26}{1560} = 0.847 \text{ N/W} \quad (20.148)$$

Also

$$f_{xVA} = \frac{F_{xk}}{S_1} = \frac{1321.26}{3000} = 0.44 \text{ N/VA} \quad (20.149)$$

Such specific thrust values are typical for tubular LIAs.

20.12 SHORT-SECONDARY DOUBLE-SIDED LIAs

In some industrial applications such as special conveyors, the vehicle is provided with an aluminum (light weight) sheet which travels guided between two layer primaries (Figure 20.29).

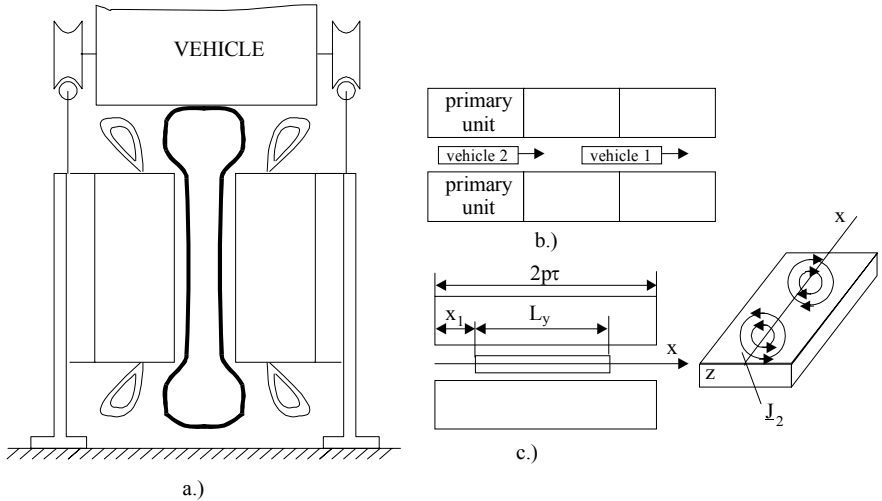


Figure 20.29 Short secondary double-sided LIA
 a.) cross section b.) multiple vehicle system c.) single vehicle

The primary units may be switched on and off as required by the presence of a vehicle in the nearby unit.

The fact that we deal with a secondary which is shorter than the primary leads to a redistribution of secondary current density along the direction of motion. Let us consider the simplified case of a single short vehicle inside a single longer primary.

The second current density paths (Figure 20.29c) reveal the transverse edge effect which may be accounted for as in previous paragraphs. Also the skin effect may be considered as done before in the chapter.

The dynamic longitudinal end effect is considered negligible. So the current density in the secondary is simply having the conventional travelling component and a constant which provides for the condition

$$\int_0^{L_v} (\underline{J}_{2z} dx) = 0 \quad (20.150)$$

$\underline{J}_{2z}$ (from (20.76) with $A_t = B_t = 0$) is

$$\underline{J}_{2z} = \frac{-jSG_e J_m e^{-j\frac{\pi}{\tau}x}}{d_{Al}(1+jSG_e)} + A \quad (20.151)$$

From (20.150)-(20.151) $\underline{A}$ is

$$\underline{A} = \frac{-jSG_e J_m}{d_{Al}(1+jSG_e)} \frac{\tau}{j\pi} \frac{\left(1 - e^{j\frac{\pi}{\tau}L_v}\right)}{L_v} \quad (20.152)$$

In fact $\underline{A}$ represents an alternative current density which produces additional losses and a pulsation in thrust. Its frequency is the slip frequency Sf_1 , the of entire $\underline{J}_{2z}$.

As expected when $L_v = 2K\tau$, $\underline{A} = 0$.

Care must be exercised when defining the equivalent parameters of the equivalent circuit as part of the primary “works” on empty airgap while only the part with the vehicle is active. 3D FEM approaches seem most adequate to solve this case.

Linear induction pumps are also complex systems characterised by various phenomena. For details see Reference 5, Chapter 5.

20.13 LINEAR INDUCTION MOTORS FOR URBAN TRANSPORTATION

LIMs when longitudinal effects have to be considered are typically applied to people movers on airports (Chicago, Dallas-Fortworth), in metropolitan areas (Toronto, Vancouver, Detroit) or even for interurban transportation. Only single sided configurations have been applied so far.

As in previous paragraphs, we did analyse all special effects in LIMs including the optimal goodness factor definition corresponding to zero longitudinal end effect force at zero slip ([Figure 20.20](#)). We will restrict ourselves here to discuss only some design aspects through a rather practical numerical example. More elaborated optimization design methods including some based on FEM are reported in [15-17]. However, they seem to ignore the transverse edge effect and saturation and eddy currents in the secondary back iron.

Specifications

- Peak thrust at standstill $F_{xk} = 12.0\text{kN}$

- Rated speed; $U_n = 34\text{m/s}$
- Rated thrust, at U_n $F_{xn} = 3.0\text{kN}$
- Pole pitch $\tau = 0.25\text{m}$ (from 0.2 to 0.3m)
- Mechanical gap, $g_m = 10^{-2}\text{m}$
- Starting primary current $I_k = 400\text{-}500\text{A}$

Data from past experience

- The average airgap flux density $B_{gn} = 0.25\text{-}0.40\text{T}$ for conductor sheet on solid iron secondary and $B_{gn} = 0.35\text{-}0.45\text{T}$ for ladder type secondary.
- The primary current sheet fundamental $J_{mk} = 1.5\text{-}2.5 \cdot 10^5\text{A/m}$.
- The effective thickness of aluminum (copper) sheet of the secondary $d_{Al} = (4\text{-}6) \cdot 10^{-3}\text{m}$.
- The pole pitch $\tau = 0.2\text{-}0.3\text{m}$ to limit the secondary back iron depth and reduce end connections of coils in the primary winding.
- The frequency at standstill and peak thrust should be larger than $f_{isc} = 4\text{-}5\text{Hz}$ to avoid large vibration and noise during starting.
- The primary stack width/pole pitch $2a/\tau = 0.75\text{-}1$. The lower limit is required to reduce the too large influence of end connection losses, and the upper limit to obtain reasonable secondary costs.

Objective functions

Typical objective functions to minimise are

- Inverse efficiency $F_1 = 1/\eta$
- Secondary costs $F_2 = C_{sec}$
- Minimum primary weight $F_3 = G_1$
- Primary KVA $F_4 = S_{lk}$
- Capitalised cost of losses $F_c = (P_{loss})_{av}$

A combination of these objectives may be used to obtain a reasonable compromise between energy conversion performance and investment and loss costs. [15]

Typical constraints

- Primary temperature $T_1 < 120^\circ$ (with forced cooling)
- Secondary temperature in stations during peak traffic hours T_2
- Core flux densities $< (1.7\text{-}1.9)\text{T}$
- Mechanical gap $g_m \geq 10^{-2}\text{m}$

Some of the unused objective functions may be taken as constraints.

Typical variables

Integer variables

- Number of poles $2p_1$
- Slots per pole per phase q
- Number of turns per coils n_c ($W_1 = 2p_1 q n_c$)

Real variables

- Pole pitch τ
- Airgap g_m
- Aluminum thickness d_{Al}
- Stack width $2a$

- Primary slot height h_s
- Primary teeth flux density B_{t1}
- Back iron tangential flux density B_{xin}
- Rated frequency f_1

The analysis model

The analysis model has to make the connection between the variables, the objective functions and the constraints. So far in this chapter we have developed a rather complete analytical model which accounts for all specific phenomena in LIMs such as transverse edge effect, saturation and eddy currents in the secondary back iron, airgap leakage, skin effect in the aluminum sheet of secondary and longitudinal end effect.

So, from this point of view, it would seem reasonable to start with a feasible variable vector and then calculate the objective function(s) and constraints and use a direct search method to change the variables until sufficient convergence of the objective function is obtained (see Hooks–Jeeves method presented in Chapter 18).

To avoid the risk of being trapped in a local minimum, a few starting variable vectors should be tried. Only if the final variable vector and the objective function minimum are the same in all trials, we may say that a global optimum has been obtained. Finding a good starting variable vector is a key factor in such approaches.

On the other hand, Genetic Algorithms are known for being able to reach global optima as they start with a population of variable vectors (chromosomes)– see Chapter 18. Neural network-FEM approaches have been presented in [16].

To find a good initial design, the methodologies developed for rotary IM still hold but the thrust expression includes an additional term to account for longitudinal effect.

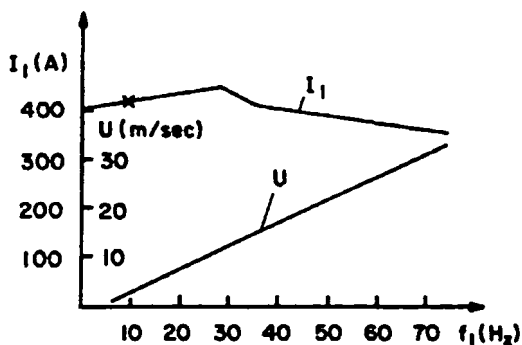


Figure 20.30 Imposed current and speed dependence of frequency

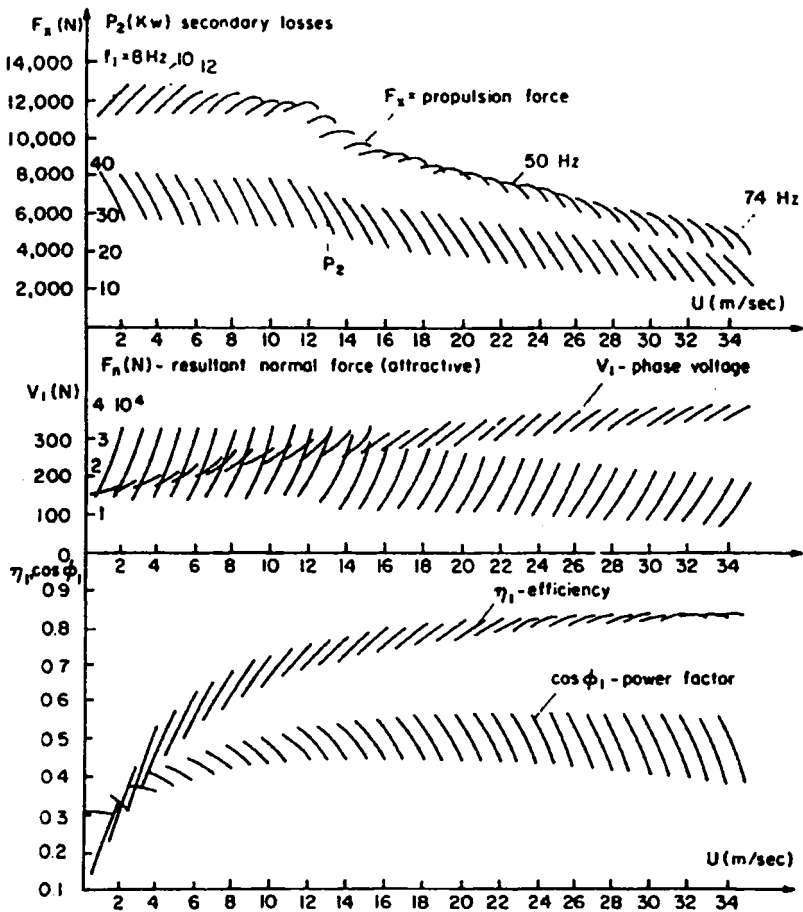


Figure 20.31 Motoring performance

Following the analysis model developed in this chapter, the following results have been obtained for the above initial data $\tau = 0.25$ m, $2p_1 = 10$, $W_1 = 90$, $I_{isc} = 420$ A (for peak thrust), $f_{isc} = 6$ Hz, $2a = 0.27$ m, $V_{isc} = 220$ V (at start), phase voltage at 20 m/s and rated thrust is $V_{1n} = 200$ V, efficiency at rated speed and thrust $\eta_1 = 0.8$, rated power factor $\cos\phi_1 = 0.55$. Performance characteristics are shown in [Figures 20.30-20.32](#).

Discussion of numerical results

- Above 80% of rated speed (34 m/s) the efficiency is quite high 0.8.
- The power factor remains around 0.5 above 50% of rated speed.
- The thrust versus speed curves satisfy typical urban traction requirements.
- Regenerative braking, provided the energy retrieval is accepted through the static converter, is quite good down to 25% of maximum speed. More

regenerative power can be produced at high speeds by adequate control (full flux in the machine) but at higher voltages in the d.c. link.

- Though not shown, a substantial net attraction force is developed (2-3 times or more the thrust) which may produce noise and vibration if not kept under control.
- These results may be treated as an educated starting point in the optimisation design. Minimum cost of secondary objective function would lead to a not so wide stack ($2a < 0.27\text{m}$) with a lower efficiency in the motor. The motor tends to be larger. The pole pitch may be slightly smaller. [15]

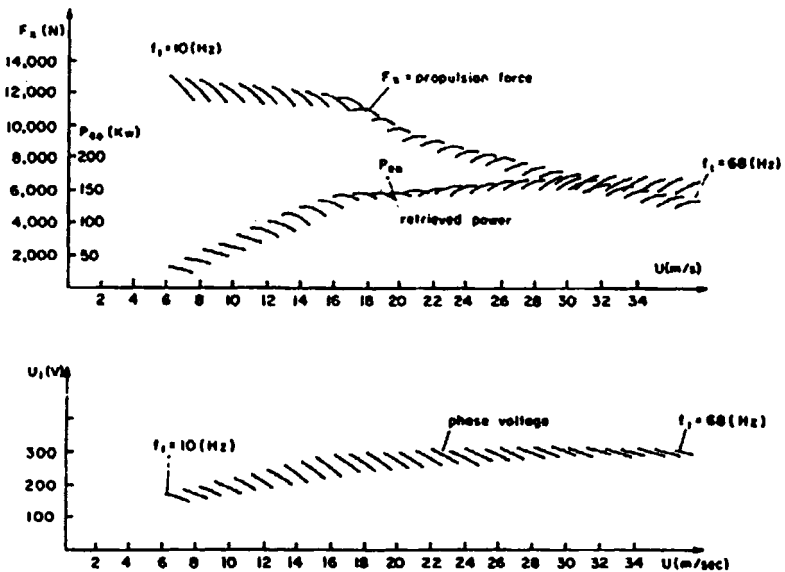


Figure 20.32 Generator braking performance

- The thermal design of secondary, especially around the stop stations, during heavy traffic, is a particular constraint which needs a special treatment. [5, pp. 248 - 250]

20.14 TRANSIENTS AND CONTROL OF LIMs

In the absence of longitudinal end effects, the theory of transients as developed for rotary induction motors may be used. The influence of transverse edge effect and secondary eddy currents may be modelled by an equivalent double cage (Chapter 13). The ladder secondary with broken bars, may need a bar to bar simulation as done for rotary IMs (Chapter 13).

A fictitious ladder [18] (or dq pole by pole [19]) model of transients may be used to account for longitudinal end effects.

Also FEM-circuit coupled models have been introduced recently for the scope. [17] The control of LIMs by static power converters is very similar to that presented in Chapter 8 for rotary IMs. Early control systems in use today apply slip frequency control in PWM-voltage source inverters such as in the UTDC2 system. [20]

Flux control may be used to tame the normal force while still providing for adequate flux and thrust combinations for good efficiency over the entire speed range.

Advanced vector control techniques-such as direct torque and flux control (DTFC) have been proposed for combined levitation-propulsion of a small vehicle for indoor transport in a clean room. [21]

20.15 ELECTROMAGNETIC INDUCTION LAUNCHERS

The principle of electromagnetic induction may be used to launch by repulsion an electrically conducting armature at a high speed by quickly injecting current in a primary coil placed in its vicinity (Figure 20.33).

A multicoil arrangement is also feasible for launching a large weight vehicle.

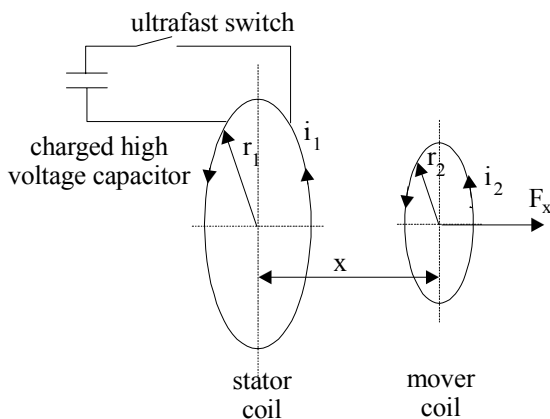


Figure 20.33 Coaxial filamentary coil structure

The principle is rather simple. The “air core” stator coil gets a fast injection of current i_1 from a high voltage capacitor through a very fast high voltage switch. The time varying field created produces an e.m.f. and thus a current i_2 in the secondary (mover) coil(s). The current in the mover coil i_2 decreases slowly enough in time to produce a high repulsion force on the mover coil. That is to launch it.

A complete study of this problem requests a 3D FEM eddy current approach. As this is time consuming, some analytical methods have been introduced for preliminary design purposes.

The main assumption is that the two currents (m.m.f.s) are constant in time during the launching process and equal to each other $W_1 I_1 = - W_2 I_2$, as in an ideal short-circuited transformer.

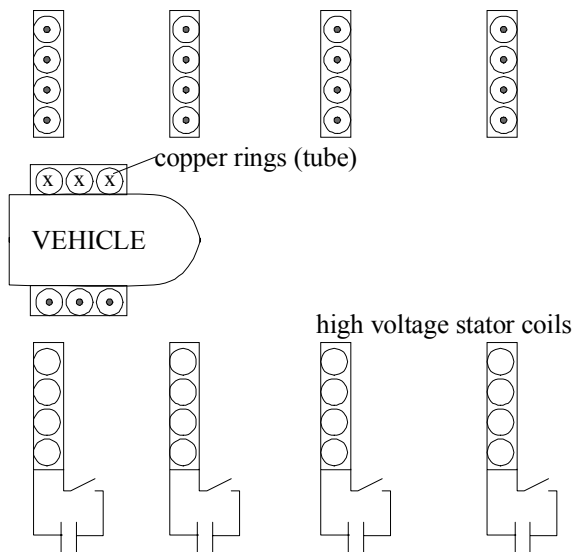


Figure 20.34 Multiple coil structure

In this case, the force F_x in the mover may be calculated from the known formula.

$$F_x = i_1 i_2 \frac{\partial M(x)}{\partial x} \quad (20.153)$$

Now the problem retorts to the computation of the mutual inductance $M(x)$ between stator and mover coil versus position.

For concentric filamentary coils, analytical expressions for $M(x)$ making use of elliptic functions exist. [22] For more involved configurations (Figure 20.34) again FEM may be used to calculate the mutual inductance M and, for given constant currents, the force F_x , even the force along radial direction which is related to motion stability. With known (given) currents the FEM computation effort is reasonably low.

The design of such a system faces two limitations”

- the numerical maximum stress on the stator coils, in the radial direction mainly, σ_s
- the mover coil maximum admitted temperature T_m .

For given stator coil base diameter D_i , mover coil weight m_α (total mover weight is $m(1 + \alpha)$) and barrel length L , the maximum speed of the mover is pretty much determined for given σ_s and T_m constraints. [22]

There is rich literature on this subject (see IEEE Trans. vol. MAG, no.1, 1999, 1997, 1995—the symposium on electromagnetic launch technology (EML)).

The radial force in concentric coils may be used to produce “magnetic” compression of magnetic powders, for example, with higher permeability, to be used in permanent magnetic electric motor with complex geometry.

20.16 SUMMARY

- Linear induction motors (LIMs) develop directly an electromagnetic force along the travelling field motion in the airgap.
- Cutting and unrolling a rotary IM leads to a single sided LIM with a ladder type secondary.
- A three phase winding produces basically a travelling field in the airgap at the speed $U_s = 2\tau f_1$ (τ -pole pitch, f_1 -primary frequency).
- Due to the open character of the magnetic circuit along the travelling field direction, additional currents are induced at entry and exit ends.
- They die out along the active part of LIM producing additional secondary losses, and thrust, power factor and efficiency deterioration. All these are known as longitudinal end effects.
- The longitudinal end effects may be neglected in the so-called low speed LIMs called linear induction actuators.
- When the secondary ladder is replaced by a conducting sheet on solid iron, a lower cost secondary is obtained. This time the secondary current density has longitudinal components under the stack zone. This is called transverse edge effect which leads to an equivalent reduction of conductivity and an apparent increase in the airgap. Also the transverse airgap flux density is nonuniform, with a minimum around the middle position.
- The ratio between the magnetization reactance X_m and secondary equivalent resistance R_2' is called the goodness factor $G_e = X_m/R_2'$. Airgap leakage, skin effect, and transverse edge effects are accounted for in G_e .
- The longitudinal end effect is proven to depend only on G_e , number of poles $2p_1$ and the value of slip S .
- The longitudinal effect introduces a backward and a forward travelling field attenuated wave in the airgap. Only the forward wave dies out slowly and, when this happens along a distance shorter than 10% of primary length, the longitudinal end effect may be neglected.
- Compensation windings to destroy longitudinal effect have not yet proved practical. Instead, designing the LIM with an optimum goodness factor G_0 produces good results. G_0 has been defined such as the longitudinal end effect force at zero slip be zero. G_0 increases rather linearly with the number of poles.
- In designing LIAs for speed short travel applications a good design criterion is to produce maximum thrust per conductor losses (N/W) at zero

speed. This condition leads to $(G_e)_s = 1$ and it may be met with a rather low primary frequency $f_{isc} = 6\text{--}15$ Hz for most applications.

- In designing LIA systems the costs of primary + secondary + power converter tend to be a more pressing criterion than energy conversion ratings which are, rather low generally for very low speeds ($\eta_1 \cos \phi_1 < 0.45$).
- The ladder secondary leads to magnetic airgaps of 1 mm or so, for short travels (in the meter range) and thus better energy conversion performance is to be expected ($\eta_1 \cos \phi_1$ up to 0.55).
- Tubular LIMs with disk shape laminations and secondary copper rings in slots are easy to build and produce satisfactory energy conversion ($\eta_1 \cos \phi_1$ up to 0.5) for short travels (below 1m long) at rather good thrust densities (up to 1 N/W and 1.5 N/cm²).
- LIMs for urban transportation have been in operation for more than a decade. Their design specifications are similar to a constant torque + constant power torque speed envelope rotary IM drive. The peak thrust at standstill is obtained at $G_e = 1$ for a frequency in general larger than (5–6)Hz to avoid large vibrational noise during starting.

For base speed (continuous thrust at full voltage) and up to a maximum speed (at constant power) energy conversion (η , KVA) or minimum costs of primary and secondary or primary weight objective functions are combined for optimization design. All deterministic and stochastic optimisation methods presented in Chapter 18 for rotary IMs are also suitable for LIMs.

- Electromagnetic induction launchers based on the principle of repulsion force between opposite sign currents in special single and multistator ring-shape stator coils fed from precharged high voltage capacitors and concentric copper rings, may have numerous practical applications. They may be considered as peculiar configurations of air core linear induction actuators. [22, 23] Also they may be used to produce huge compression stresses for various applications.

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